INELASTIC BUCKLING ANALYSIS OF ALUMINIUM SHELLS

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ABSTRACT

The paper summarises a part of the activity of CEN/TC250 SC9 Committee, devoted to the preparation of Eurocode 9 “Design of Aluminium Structures”. The results of a wide imperfection sensitivity F.E.M. analysis, dealing with aluminium cylinders subjected to axial load, uniform external pressure and torsion, are discussed. Numerical simulations have been carried out by accounting for a wide geometrical imperfection spectrum, in order to consider the most dangerous distributions. Results of almost 6000 F.E.M. simulation runs have been used to delineate a numerical data-set for the definition of buckling curves for aluminium alloys shells, to be introduced into the new part prEN1999-1-5 of Eurocode 9. For the sake of homogeneity, the basic layout of prEN1993-1-6 has been referred to as a general framework. Nevertheless, it is shown that buckling curves given in EC3 can not be used for aluminium shells, but require proper modification.

Key Words: Shell structures, aluminium alloys, buckling, imperfection sensitivity

1. INTRODUCTION

Investigation on the buckling of aluminium alloy shells started in the Thirties, along with the great development of aircraft industry and with the definitive substitution of obsolete lattice structures with fully metal plated structures. Compared with trussed structures, in fact, shell structures exhibited higher stiffness and lower weight and, hence, improved structural performance in highly demanding applications. The use of aluminium alloys in thin walled structures, on the other hand, put into evidence not only the well known imperfection sensitivity of shells, but also the great importance of pre-buckling inelastic deformations and of their influence on both imperfection sensitivity and post-critical response of the shell. As a consequence, it was soon evident that the buckling of aluminium shells could not be faced without taking into proper account all material features, which make aluminium alloys very different from steel and other metals.

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In the field of civil applications, due to averagely higher values of shell wall thickness, the problem of pre-buckling inelastic behaviour was even more important, and this required a radically different approach to buckling problems, also from the codification point of view. Unfortunately, it was not before the advent of refined F.E.M. calculation techniques, namely at the end of the Seventies, that was possible to frame the twofold problem of inelastic buckling and imperfection sensitivity of shells in an exhaustive way. Research carried out in aerospace field, in fact, even though very accurate and extensive, was mainly concerned with very thin shells, which failed in purely elastic range and, therefore, were relatively material-independent.

Alongside with the improvement of F.E.M. calculation procedures, that highlighted the strong interaction that may occur between imperfection and plasticity effects, increasing applications of shells structures in the civil field demanded for proper codification. First issues in this context were a concern of the oil industry, which was deeply interested in the design of storage tanks and similar structures. Afterward, a steadily increasing interest in shells structures for transportation and general purpose applications pushed both national and international institutions (e.g. BS, DIN, ECCS) to produce the first codification on steel shells. Notwithstanding, aluminium shells were kept outside the mainstream codification activity until the end of the Eighties, when the Italian UNIMET Committee, chaired by F.M. Mazzolani, presented a first proposal of codification on the buckling of aluminium alloy shells. The proposal was modelled on the basis of ECCS Recommendations on steel shells, with suitable modifications in order to take into account the peculiar features of light alloys.

During the Nineties, the drafting of structural Eurocodes, as a matter of fact, inhibited the development of codification issues at national level, which is why the Italian proposal, and hence the codification on aluminium shells, came at a stand-by point. Only in the latest years, within the activity of CEN/TC250 SC9 Committee (chairman F.M. Mazzolani), the new Part 1-5 “Supplementary rules for shell structures” (prEN1999-1-5) of Eurocode 9 “Design of aluminium structures” has been accepted for being included into the general Eurocode programme. This part, which is presently in progress, represents the very first codification issue at European level dealing with aluminium alloy shells and, for this reason, it deserves particular attention. The document is globally based on the same layout of the corresponding EC3 Part 1-6 on steel shells (prEN1993-1-6) [1,2], but with proper variations in order to take into account inherent aspects of aluminium alloys, in particular as far as buckling problems are concerned.

This paper wants to summarise the extensive work carried out at both Universities of Naples by the research team led by F.M. Mazzolani in the field of aluminium shells, with particular emphasis to the support given to the codification activity. Results of recent F.E.M. simulation analyses dealing with aluminium cylinders subjected to axial load, uniform external pressure and torsion are discussed with the purpose to highlight the particular aspects of inelastic buckling and imperfection sensitivity of aluminium shells. The investigation, relying on almost 6000 F.E.M. simulation runs, has been carried out by accounting for a wide geometrical imperfection spectrum, in order to consider the most dangerous distributions. Results have then been exploited to define a numerical data-set for the set-up of buckling curves for aluminium alloys shells, to be proposed for the introduction into the prEN1999-1-5.

2. THE STATE OF CODIFICATION ON ALUMINIUM SHELLS

In spite of the great effort carried out in the field of codification in the recent years, the problem of buckling of shell structures is not yet definitively assessed. This may be considered as a consequence, on one hand, of the difficulty to evaluate the very high imperfection sensitivity of such structures, on the other hand, of the problem to take into account the effect...
of material plasticity. As a consequence of this, the effect of imperfection is still evaluated through the traditional, empirical "Lower Bound Design Philosophy", according to which a knock-down factor of buckling loads, usually denoted by $\alpha$, is introduced in order to set a lower limit of the scattered experimental data. In the most up-to-dated codes, namely ESDU (U.K. 1974), MI-04.184 (Hungary, 1978), AISC (Australia, 1983), ECCS (Europe, 1988) and EC3 (Europe, 2004), the factor $\alpha$ is related to the imperfection magnitude according to shell quality classes, but without considering the actual imperfection distribution. Similarly, the effect of material plasticity is only approximately considered in the codes, usually by means of an empirical regression curve, which is not able to allow for the different inelastic features of materials. Moreover, all codes, except the new Part 1-5 of EC9, are mostly concerned with mild steel shells, whereas no specific allowance is made for round-house-type materials, such as high strength steels and aluminium alloys, whose behaviour is peculiarly hardening.

A first attempt to adapt the EC3 approach to aluminium shells was proposed by Mazzolani & Mandara [3,4,5], by modifying the $\lambda_0$, $\beta$ and $\eta$ parameters provided in the piecewise formulation of buckling factor $\chi$ given in prEN1993-1-6. Nevertheless, as shown in Section 5, this led to an unjustified excess of conservativeness and, most of all, to a lack of accuracy in the interpretation of buckling data in the elastic-plastic region. In addition, this approach would involve the commonly recognised difference between strong and weak hardening alloys to be completely missing. In order to overcome such limits, an alternative formulation for aluminium shell buckling curves has been presented for the same load cases as considered in prEN1993-1-6 [3,6]. The proposal is based on the format already adopted for the buckling of aluminium members in compression and codified into prEN1999-1-1. Imperfection reduction factors are kept equal to those provided into prEN1993-1-6, except for the case of axial compression. The characteristic buckling strengths are obtained by multiplying the characteristic limiting strength $f_{0.2}$ by buckling reduction factors $\chi$. The buckling reduction factors $\chi_x$, $\chi_0$ and $\chi_t$ are expressed as a function of the relative slenderness $\lambda$ of the shell by means of the relationship $\chi = \alpha \chi_{\text{perf}}$, in which $\alpha$ is the imperfection reduction factor, depending on the load case, shell slenderness and quality class, and $\chi_{\text{perf}}$ is the buckling factor for a perfect shell, given by:

$$\chi_{\text{perf}} = \left[ \frac{\phi + \sqrt{\phi^2 - \lambda^2}}{2} \right] \quad \text{with} \quad \phi = 0.5 \left[ 1 + \alpha_0 \left( \lambda - \lambda_0 \right) + \lambda^2 \right] \quad (1)$$

where $\lambda_0$ is the squash limit relative slenderness and the parameter $\alpha_0$ depends on the alloy.

Apart for the value of the shell imperfection reduction factor $\alpha$, Equation (1) is formally identical to the one set out in prEN1999-1-1 for aluminium members in compression. In particular, for both external pressure (circumferential compression) and torsion (shear) the same factors $\alpha_0$ and $\alpha_t$ given in prEN1993-1-6 have been kept [6]. Due to the different imperfection sensitivity exhibited by aluminium cylinders when they buckle in plastic range, a slightly different expression of the factor $\alpha_x$ has been proposed for axially loaded cylinders, that is:

$$\alpha_x = \frac{1}{1 + 2.60 \left( \frac{1}{Q} \left[ \frac{0.6E}{f_{0.2} \sqrt{\lambda_x - \lambda_{x,0}}} \right] \right)^{1.44}} \quad (2)$$

Furthermore, the same distinction in quality classes A, B and C as in prEN1993-1-6 has been kept, which results in keeping the same values of both imperfection limits and quality parameter $Q$ in case of axially loaded cylinders.

3. THE F.E.M. SIMULATION
The buckling behaviour of cylinders is dominated by their high imperfection sensitivity, due to the asymmetric stable-unstable response at bifurcation point, which usually results in a remarkable reduction of buckling loads compared to theoretical predictions, depending on both load case and imperfection magnitude. This aspect can be very significant also in case of thick shells failing in plastic range ($R/t < 200$), where an interaction between imperfections and plasticity effect can occur, with a strong influence on the buckling modes and postcritical behaviour. This effect is particularly evident in case of axially loaded cylinders and is a relevant issue of aluminium alloy shells because of the hardening behaviour of material.

For this reason, the analysis has been mostly concerned with cylinders failing by elasto-plastic buckling. In the same way, alloys have been selected that allow for a full exploitation of material strain hardening behaviour [3]. They have been chosen according to the well accepted distinction between Weak Hardening alloys and Strong Hardening alloys, already introduced in the European Recommendations on Aluminium Alloys of ECCS (1978), and also considered in Eurocode 9.

The buckling analysis has been performed via static F.E.M. analysis, aimed at the evaluation of both shell limit load and postcritical response as a function of the initial imperfection distribution. As shown by Mandara & Mazzolani [8,9], such a kind of approach is suitable when no snap-through or mode jumping is expected, as usually happens in case of relatively thick cylinders which buckle in plastic range. The F.E.M. analysis has been carried out with ABAQUS code [10] using the RIKS option for the solution algorithm, the DEFORMATION PLASTICITY option for material law and four-node, reduced integration S4R5 shell elements. The ABAQUS option DEFORMATION PLASTICITY is based on a pluriaxial formulation of the classical Ramberg-Osgood law and, thus, is particularly suitable to aluminium alloys. The exponent $n_{R.O.}$ of the Ramberg-Osgood material law has been evaluated according to the Steinhardt proposal ($n_{R.O.} = \left(\frac{f_{0.2}}{10}\right)$, with $f_{0.2}$ expressed in N/mm² [7].

The imperfection model assumed in the analysis is:

$$w = w_0 \sum e^{-k_1(x-x_0)^2} \cos \left( k_{2x} \pi \frac{(x-x_0)}{L} \right) e^{-k_1(y-y_0)^2} \cos \left( k_{2y} \frac{(y-y_0)}{R} \right)$$

By giving suitable values to $k_{1x}$, $k_{1y}$, $k_{2x}$, $k_{2y}$, $x_0$ and $y_0$, Equation (3) can describe imperfection distributions similar to either axisymmetric or asymmetric critical and postcritical modes, as well as localised surface defects. The model can be used for interpreting the imperfection affecting relatively thick shells (Mandara [8], Mandara & Mazzolani [9]). In most cases, an initial imperfection distribution similar to single or multiple critical modes has been assumed in the analysis, by giving the parameters $k_{2x}$ and $k_{2y}$ a value corresponding to the number of longitudinal ($m$) and circumferential ($n$) waves at buckling, respectively. In this way the most severe condition for the buckling response has been investigated, so as to determine a lower bound of the ultimate load carrying capacity as a function of the initial imperfection magnitude. Other types of localised imperfection (say a concentrated dimple placed at cylinder midlength or a continuous longitudinal groove), obtained by giving parameters $k_{1x}$, $k_{1y}$, $k_{2x}$, $k_{2y}$, $x_0$ and $y_0$ specific values, have been also considered. Examples of imperfection distributions according to Equation (3) are illustrated in Figure 1. A more detailed description of the imperfection distributions assumed in the analysis is given in Mazzolani & Mandara [3].

Shell imperfection patterns have been assumed according to both elastic and plastic bifurcation modes. A synopsis view of bifurcation loads, critical modes and plasticity factors is given in Table 3 for all relevant load cases considered in the analysis. In case of plastic buckling, critical axisymmetric modes derived by a bifurcation analysis has been considered. The minimum critical load in plastic range is generically expressed as $\sigma_{cr,p} = \eta \sigma_{cr,el}$, where $\eta$ is a
reduction factor introduced to take into account the effect of material plasticity obtained from literature (Gerard [11]).

![Fig. 1 – Some imperfection distributions according to Equation (1) (n = 7, m =15), (m =16), (n = 4)](image)

Table 1 – Synopsis of elastic and elasto-plastic buckling loads and corresponding critical modes for load cases under consideration

<table>
<thead>
<tr>
<th>Cylinders under axial load</th>
<th>Elastic bifurcation load</th>
<th>Number of circumferential (n) and axial (m) waves at elastic buckling</th>
<th>Number of axial (m) waves at plastic buckling</th>
<th>Plasticity reduction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{cr,el} = \frac{E}{\sqrt{3(1-\nu^2)} R}$</td>
<td>$\left(\frac{m^2 + (nL/\pi R)^2}{m^2}\right)^{\frac{1}{2}} \frac{2 L^2}{\pi R} \sqrt{3(1-\nu^2)}$</td>
<td>$m = \frac{L}{\pi R} \sqrt{\frac{12}{6 + 9 E_s + E_s/E_t}}$</td>
<td>$\eta = \sqrt{E_s E_t}$</td>
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<table>
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<tr>
<th>Cylinders under external pressure</th>
<th>Elastic bifurcation load</th>
<th>Number of circumferential (n) waves at buckling</th>
<th>Plasticity reduction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{cr,el} = \frac{Et}{R} \left[ \frac{1}{n^2 + 1 - \nu^2 + \frac{1}{2} \left( \frac{\pi R}{L} \right)^2 \frac{1}{12 R\left[1-\nu^2\right]^{\frac{3}{2}}} \left( n^2 - 1 \right) \left( \frac{\pi R}{L} \right) \right]$</td>
<td>$n = 2.7 \left( \frac{R}{L} \right)^{0.5} \left( \frac{R}{t} \right)^{0.25}$</td>
<td>$\eta = \frac{1}{4} \frac{E_s}{E} + \frac{3}{4} \frac{E_s}{E}$</td>
<td></td>
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<table>
<thead>
<tr>
<th>Cylinders under torsion</th>
<th>Elastic bifurcation load</th>
<th>Number of circumferential (n) waves at buckling</th>
<th>Plasticity reduction factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{cr,el} = 0.75 E \left( \frac{R}{L} \right)^{\frac{1}{2}} \left( \frac{t}{R} \right)^{\frac{5}{4}}$</td>
<td>$n = 4.2 \left(0.75\right)^{\frac{1}{8}} \frac{R}{L} \sqrt{\frac{R}{t}}$</td>
<td>$\eta = \frac{E_s}{E}$</td>
<td></td>
</tr>
</tbody>
</table>

4. DISCUSSION OF RESULTS

4.1 General

The F.E.M. imperfection sensitivity analysis has emphasised the strong influence of the initial imperfection pattern, in particular when directed according to critical modes. Typical imperfection sensitivity curves are plotted in Figure 2, showing the great reduction of the buckling load $P_u$ with the imperfection magnitude $w_0$, compared with the theoretical elasto-plastic critical load $P_{cr,th}$. In the evaluation of Gerard’s $\eta$ coefficients, the values of both tangent $E_t = d\sigma/d\epsilon$ and secant moduli $E_s = \sigma/\epsilon$ have been calculated by means of the Ramberg-Osgood law $\epsilon = \sigma/E + 0.002(\sigma/E)^n$ [7]. Limits corresponding to quality classes A, B and C,
as defined in prNV1993-1-6, are also drawn. In general, the effect of initial imperfection increases as $R/t$ and $L/R$ ratios, as well as the elastic limit strength $f_{0.2}$ increase, while in the same conditions the effect of plasticity is relatively smaller.

Fig. 2 – Typical imperfection sensitivity curves for cylinders under axial load (a), uniform external pressure (b) and torsion (c)

4.2 Axial load

F.E.M. analysis confirms that buckling may be either axisymmetric or asymmetric depending on the initial imperfection distribution, as well as on the value of buckling stress. For an asymmetric imperfection the typical snap-through behaviour of perfectly elastic shell tends to disappear when plasticity effects occur, at least for small imperfection magnitudes (Figure 3a,b). Transition from asymmetric (Figure 3a) to axisymmetric (Figure 3b) buckling corre-
sponds to a critical value of imperfection $w^*$ (Mandara [8], Mandara & Mazzolani [9]). When considering an axisymmetric imperfection, buckling is always symmetric with the ultimate load decreasing with the imperfection magnitude.

In case of imperfections other than those corresponding to critical modes, in general, the imperfection sensitivity for concentrated or linear defects is remarkably lower than for imperfection distributions similar to critical modes.

The influence of boundary conditions is rather significant when the buckling is axisymmetric, due to the fact that buckles take place near the loaded ends. On the contrary, it drops down when the buckling deflection is predominantly developed in shell intermediate region. This happens in case of asymmetric instability and, generally speaking, in all cases when buckling occurs without significant precritical plastic deformations.

Eventually, a great scatter of results can be observed (Figure 2a), which confirms the commonly acknowledged experimental performance of axially loaded cylinders. Such scatter increases as long as $R/t$ ratio and elastic limit $f_{0.2}$ increase. The lower bound of numerical data corresponds, with good approximation, to imperfection distributions directed according to a single or to a combination of critical modes obtained from Equation (3).

4.3 External pressure

Both theoretical and experimental predictions on the buckling pattern are fully confirmed by F.E.M. analysis. The buckled configuration always consists of one longitudinal halfwave and of a number of circumferential halfwaves increasing with the $R/t$ ratio of the cylinder (Figure 4c). This buckling pattern is kept in inelastic range, too. The imperfection sensitivity analysis has shown a lower influence of geometrical imperfections on the load bearing capacity compared with axially loaded cylinders (Figure 2b). This is confirmed by a reduced scatter of results, as well as by a softer response at buckling (Figure 3c). In addition, a relatively small sensitivity to localised defects has been detected. The effect of restraint conditions is significant for very short cylinders, only, and the theoretical prediction of the critical load is well caught in case of hinged ends. In conclusion, imperfection distributions directed according to the elastic critical mode give place, as a rule, to the lowest buckling response of the shell.

4.4 Torsion

For this load case the same considerations made for cylinders under external pressure hold. The buckling shape is well interpreted by F.E.M. analysis, showing that most of deformation is concentrated in the central region of the cylinder (Figure 4d). As in the case of cylinders subjected to external pressure, the influence of restraint condition is negligible in most cases, apart from the case of very short cylinders. F.E.M. results, however, best agree with theoretical prediction in case of hinged ends. Finally, the observed scattering of results is very reduced and, also, a slightly lower global imperfection sensitivity is observed (Figure 2c).

4.5 Semi-probabilistic analysis of results for axially loaded cylinders

Because of the great scattering observed in the buckling strength data of axially loaded cylinders, numerical F.E.M. results have been interpreted in a semi-probabilistic way, so as to extrapolate a lower value of the ultimate load, corresponding to a given fractile value (e.g. 5%). The Weibull extreme distribution $P(x) = 1 - e^{-x^\gamma}$ has been used to this aim, where $\gamma$ and
β are characteristic parameters to be fitted on the basis of available data. From the previous
equation the probability density curve can be obtained:

\[ p(x) = \frac{dP(x)}{dx} = \frac{1}{\gamma \beta^{1/\gamma}} x^{(1/\gamma-1)} e^{-(x/\beta)^{1/\gamma}} \]  

\( (4) \)

Fig. 3 – Typical load-displacement curves for some of examined cases: a) axial load, asymmetric imperfection; b) axial load, axisymmetric imperfection; c) external pressure; d) torsion
Fig. 4 – Buckling deflected shapes: a) axial load, elastic buckling; b) axial load, plastic buckling; c) external pressure; d) torsion

Fig. 5 – Semi-probabilistic exploitation of simulation data according to Weibull extreme law
Such distribution, well suited to the description of random variables ranging between 0 and 1, has been already proposed for the stochastic evaluation of the buckling load of imperfect cylinders (Mendera [12]). Parameters $\gamma$ and $\beta$ have been estimated on the basis of numerical data divided according to shell quality classes as defined in prEN1993-1-6 (Mazzolani & Mandara [3]), as a function of material, $R/t$ ratio and quality class. Some of obtained cumulative curves, in which the stochastic variable $x$ has to be assumed as the cylinder buckling load, are shown in Figure 5, together with the corresponding Weibull cumulative curves and characteristic 5% lower bound. Such limit has also been used for fitting buckling curves of axially loaded cylinders proposed for prEN1999-1-5, shown in Figure 6.

5. COMPARISON OF BUCKLING CURVES

In Figures 6 to 8 the comparison between three buckling curves is shown, namely the curve of EC3, the curve of modified EC3 with different parameters and the curve proposed for prEN1999-1-5. For the sake of comparison, some of the experimental results presented in Mandara & Mazzolani [13] on stocky extruded aluminium cylinders in compression are also shown in Figure 6. The three approaches mostly differ in the transition region between elastic and plastic buckling. The EC3 curves are clearly unconservative when applied to aluminium shells, in particular in case of cylinders under axial load and torsion, whereas they could be suitable for cylinders in torsion.
Fig. 7 – Comparison of different buckling curves liable to be proposed for EC9 with F.E.M. data – cylinders under external pressure

Cylinders under external pressure
Weak hardening alloys
Quality class B

Cylinders under external pressure
Strong hardening alloys
Quality class C

Cylinders under torsion
Weak hardening alloys
Quality class B

Cylinders under torsion
Strong hardening alloys
Quality class B
To some extent, they could be corrected using different coefficients in the curve formulation, (see “Modified EC3 curve” in Figures 6 to 8), but this would result in a worse approximation in the lower slenderness range and, however, would lead to overcome the commonly accepted difference between weak and strong hardening alloys, which is clearly underlined in EC9, too. In conclusion, curves proposed for EC9 yield the best fitting of both F.E.M. and experimental data. Also, they are homogeneous with buckling curves for aluminium members in compression given in EC9.

6. CONCLUSIONS

This paper summarises the wide background research recently carried out within the activity of CEN/TC250 SC9 Committee, chaired by F.M. Mazzolani, in the field of aluminium shells. This work aimed at issuing the new part Part 1-5 “Supplementary rules for shell structures” of EC9, which is presently in progress and should be completed by the end of 2004. To this purpose, ad-hoc buckling curves have been evaluated, fitted against the results of a wide parametric F.E.M. analysis for the load cases of axial load, external pressure and torsion. The analysis highlighted the main aspects of shell stability, with special emphasis to the case of plastic buckling. Buckling approach according to EC3 has been also discussed and compared to the one proposed for EC9. As already shown by the authors in other papers, buckling curves of EC3 are inadequate to aluminium shells, which is why a completely new set of buckling curves has been proposed for prEN1999-1-5, assuming the basic formulation already implemented in prEN1999-1-1 for aluminium members in compression. The new curves provide a closer interpretation of buckling data falling in the intermediate slenderness range. Also, they clearly underline the difference between weak and strong hardening alloys. For this reason, such curves are presently under discussion for their possible implementation into the new part prEN1999-1-5 of EC9.

7. REFERENCES


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