SKEW REINFORCEMENT DESIGN IN REINFORCED CONCRETE TWO DIMENSIONAL ELEMENTS

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Abstract

Nowadays is more and more often necessary to design two dimensional reinforced concrete elements both to satisfy architectural demands, and to answer to traffic safety requirements in the design of road and railways infrastructures.

As a consequence is constantly rising the use of finite elements analysis to model structures and to calculate internal actions. Therefore comes to attention the problem of reinforcements design, as steel bars should be, in general, displaced along not orthogonal directions.

Consequently, two different kind of design problems can be outlined: reinforcement direction choice and reinforcement ratio between the chosen directions evaluation.

Such problems can be easily overcome using a referring mechanical model consolidated in literature and written according to the ultimate plastic behaviour of the elements on which is applied an optimization technique based on genetic algorithms.

In the paper both the ultimate resisting mechanism with generic reinforcement directions and the way genetic algorithms are employed to optimize reinforcement quantity and direction are shown.

Keywords: skew, reinforcement, plates, slab

1. Introduction

For the evaluation of internal actions path within a two dimensional concrete structure the most common tool is nowadays the use of linear finite elements, like shell, slab or plate.

In the following the design of concrete shell elements will be considered in detail as the slab and plate ones are easily derivable by simplification of the previous one.

The output of a fem analysis done with shell elements is eight components of internal actions: three from the plate (n_x, n_y, n_{xy}), three from the slab (m_x, m_y, m_{xy}), two out of the plane shear components (t_x, t_y) (fig.1).

The suitable design model able to take into account this whole bunch of internal actions, derived as a lower bound solution, may be considered well established in the literature [1] [2], at least in the case of orthogonal reinforcement.

Moreover, this design model has been recently improved by the introduction of a new safety criterion for concrete working in biaxial state of stress, based on a large series of heterogeneous experimental tests [3] [4] [5].

In the following, the design model for shell elements is generalized, for whichever orientation of reinforcement layers, that are very often required in practice for instance in skew slab bridges.

Moreover a guidance tool is given for the optimum choice of reinforcement directions and/or directional reinforcement ratios in order to minimize the global amount of reinforcement.

For this second aspect the genetic algorithms shall be used, in their optimisation formulation based on the genetic mechanisms and struggle for life [6] [7].

2. Shell resisting model

Sign convention for positive internal action components is pictured in fig.1 and fig.2; reinforcement orientation and their distance "c" are illustrated in fig.3, with c_s and c_i respectively top and bottom cover to the axis of the reinforcement layers.

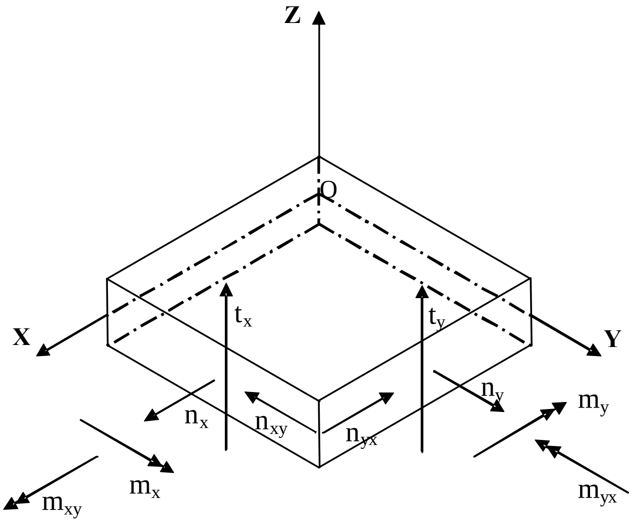

Figure 1 : Shell scheme

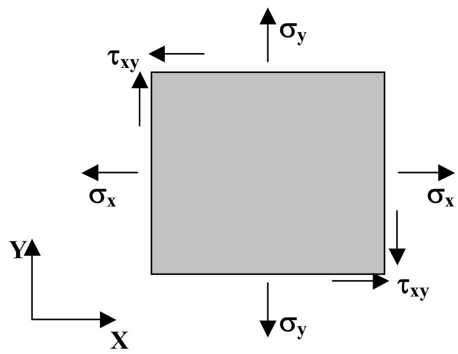

Figure 2 : Plate scheme

As first step it should be checked whether the shell is cracked or not; this verification may be performed in agreement with [8] applying the Ottosen criterion [9] [10] to different levels within the element thickness "*t*"; for this verification concrete will be considered as uncracked and the stresses consequently evalued.

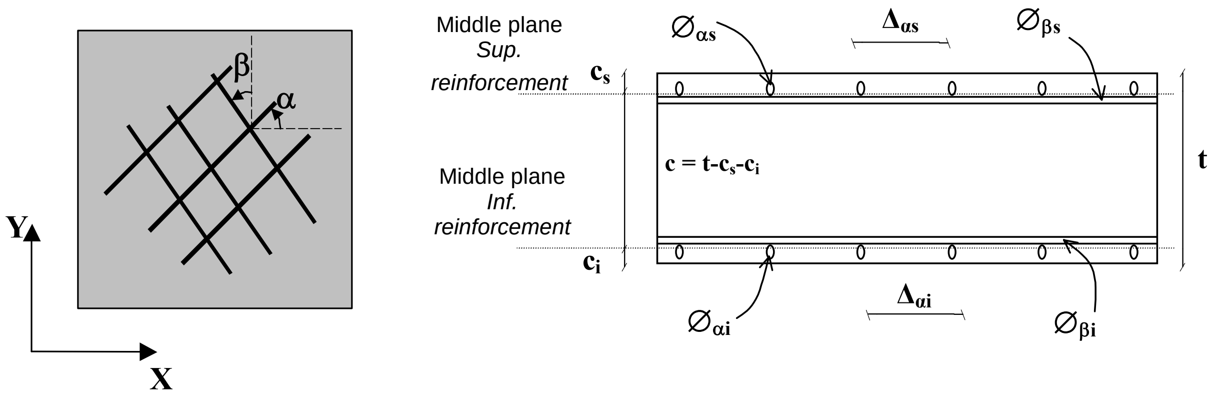

Figure 3 : Reinforcement layout

If the element results uncracked, reinforcement resisting contribution should be added to the concrete one taking into consideration the deformation state in the reinforcement directions.

On the contrary, if it results to be cracked the sandwich model will be used: two external plates should be individuated, able to carry only membrane forces, and an internal layer able to transfer the out of the plane shear components.

The internal layer will be designed, in analogy with the beam approach, along the shear principal direction, individuated by $tan(\varphi_o)= t_y/t_x$, considering the principal shear $t_o = sqrt(t_x^2 + t_y^2)$, where φ_o is the deviation of t_o with respect to the x axis. If a specific shear reinforcement is required and then a truss model is established along the shear principal direction, the following additional membrane forces coming from the truss model must be taken into account within the external plates:

$$n_x = \frac{t_x^2}{t_o}\cot\theta_t \quad ; \quad n_y = \frac{t_y^2}{t_o}\cot\theta_t \quad ; \quad n_{xy} = \frac{t_x t_y}{t_o}\cot\theta_t \tag{1}$$

where θ_t is the stress field due to principal shear inclination respect to the mean plane of the shell. In the external plates, once established a tentative value for the corresponding thickness, the internal actions may be evaluated by simple equilibrium conditions as (fig.4)

$$Top\begin{cases} n_{xs} = n_x\dfrac{z-z_s}{z} + \dfrac{m_x}{z}\left(+\dfrac{t_x^2}{2t_0}\cot\theta_t\right) \\ n_{ys} = n_y\dfrac{z-z_s}{z} + \dfrac{m_y}{z}\left(+\dfrac{t_y^2}{2t_0}\cot\theta_t\right) \\ n_{xys} = n_{xy}\dfrac{z-z_s}{z} + \dfrac{m_{xy}}{z}\left(+\dfrac{t_x t_y}{2t_0}\cot\theta_t\right) \end{cases} \quad Bottom\begin{cases} n_{xi} = n_x\dfrac{z-z_i}{z} - \dfrac{m_x}{z}\left(+\dfrac{t_x^2}{2t_0}\cot\theta_t\right) \\ n_{yi} = n_y\dfrac{z-z_i}{z} - \dfrac{m_y}{z}\left(+\dfrac{t_y^2}{2t_0}\cot\theta_t\right) \\ n_{xyi} = n_{xy}\dfrac{z-z_i}{z} - \dfrac{m_{xy}}{z}\left(+\dfrac{t_x t_y}{2t_0}\cot\theta_t\right) \end{cases} \tag{2}$$

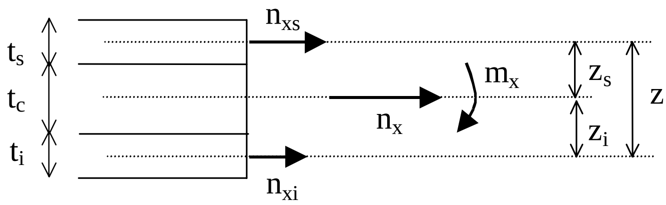

Figure 4 : plates internal actions

where the contributions within brackets should be considered only if a truss model is established.

The process is now reconducted to the design of a plate with two order of skew reinforcement.

3. Plate resisting model

For the description of biaxial concrete behaviour, in agreement with [8], the Kupfer and Gerstle proposal is considered [11]. In practice three regions for the behaviour definition of plates may be individuated (fig.5):

- In region 0 concrete is in biaxial compression state and the plate capacity may be enhanced adding the contribution of compressed reinforcement;
- In region 2 concrete is able to carry the internal actions and only the minimum reinforcement for the control of unforeseen cracking is required;
- In region 1 concrete is cracked and reinforcement is necessary for the equilibrium.

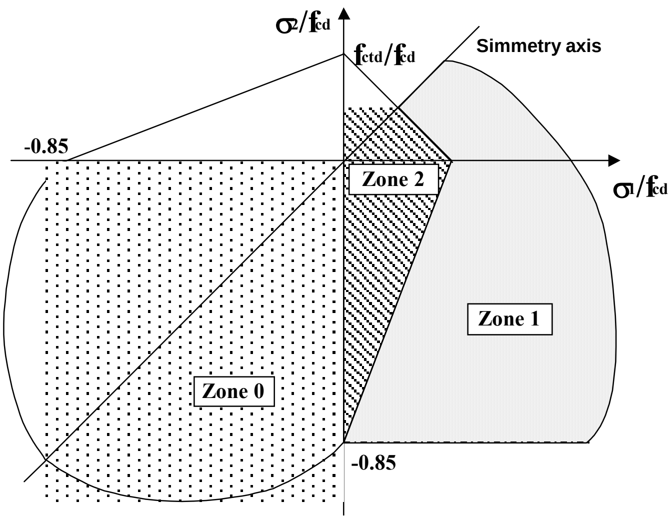

Figure 5 : Concrete failure surface

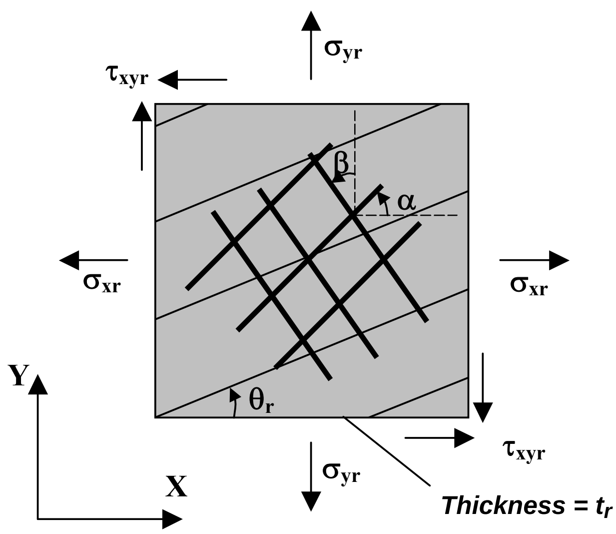

Figure 6 : Plates conventions

Then, with reference to fig.6, in which $\sigma_{xr}= n_{xr}/t_r$, $\sigma_{yr}= n_{yr}/t_r$, $\tau_{xyr}= n_{xyr}/t_r$, and t_r *(r = s,i)* is the plate thickness, may be considered a section of the element with a plane parallel to the direction of the ultimate stress field in concrete (θ) (fig.7); the following equilibrium equations may be established:

$$\sigma_{xr}\ sin\,\theta_r + \tau_{xyr}\ cos\,\theta_r - \rho_{\alpha r}\sigma_{s\alpha r}a_r\ cos\,\alpha - \rho_{\beta r}\sigma_{s\beta r}b_r\ sin\,\beta = 0 \qquad (3)$$

$$-\sigma_{yr}\ cos\,\theta_r - \tau_{xyr}\ sin\,\theta_r - \rho_{\alpha r}\sigma_{s\alpha r}a_r\ sin\,\alpha + \rho_{\beta r}\sigma_{s\beta r}b_r\ cos\,\beta = 0 \qquad (4)$$

where $\rho_{\alpha r}$ and $\rho_{\beta r}$ are the geometrical reinforcement ratios respectively in directions α and β in reinforcement layer *r*.

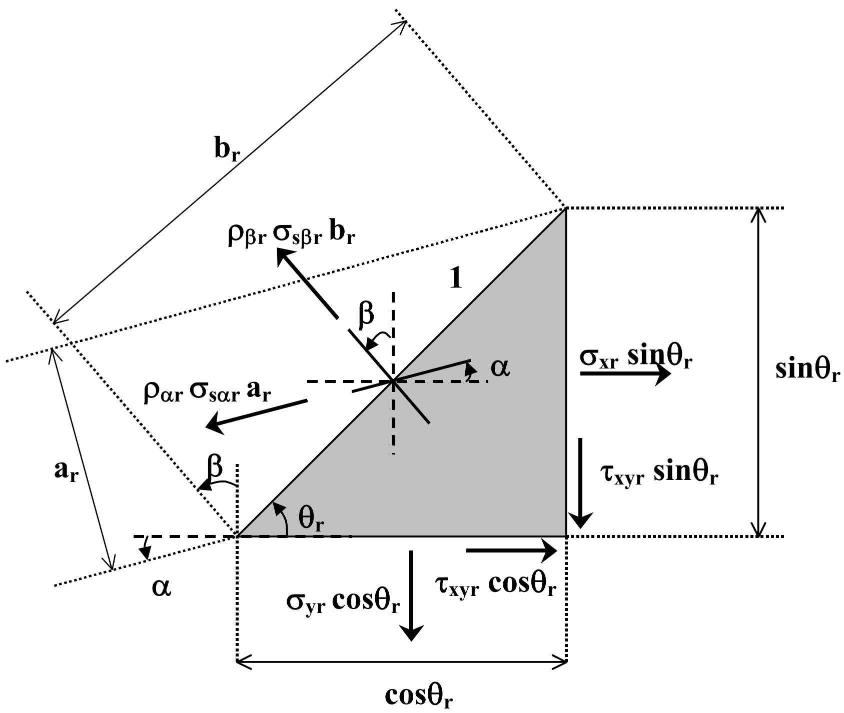

Figure 7 : Equilibrium of the section parallel to the compression field

Adding now eq (3) multiplied by *cos*β to equation (4) multiplied by *sin*β, $\rho_{\alpha r}\sigma_{s\alpha r}$ may be derived as:

$$\rho_{\alpha r}\sigma_{s\alpha r} = \frac{\sigma_{xr}\, sin\theta_r\, cos\beta - \sigma_{yr}\, cos\theta_r\, sin\beta + \tau_{xyr}\, cos(\theta_r + \beta)}{sin(\theta_r - \alpha)cos(\alpha - \beta)} \tag{5}$$

and similarly, deducing equation (4) multiplied by cosα from equation (3) multiplied by sinα, we obtain

$$\rho_{\beta r}\sigma_{s\beta r} = \frac{\sigma_{xr}\sin\theta_r \sin\alpha + \sigma_{yr}\cos\theta_r \cos\alpha + \tau_{xyr}\sin(\theta_r + \alpha)}{\cos(\theta_r - \beta)\cos(\alpha - \beta)} \tag{6}$$

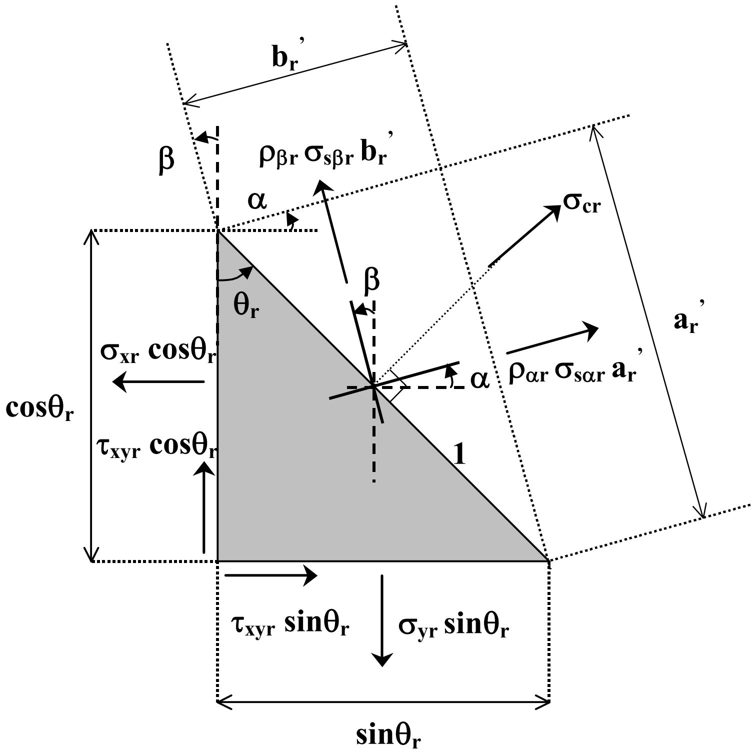

Figure 8 : Equilibrium of the section orthogonal to the compression field

Considering now a section of the element with a plane orthogonal to the stress field in concrete (fig.8) the following equilibrium equations may be established

$$-\sigma_{xr}\, cos\theta_r + \tau_{xyr}\, sin\theta_r + \rho_{\alpha r}\sigma_{s\alpha r} a'_r\, cos\alpha - \rho_{\beta r}\sigma_{s\beta r} b'_r\, sin\beta + \sigma_{cr}\, cos\theta_r = 0 \tag{7}$$

$$-\sigma_{yr}\, sin\theta_r + \tau_{xyr}\, cos\theta_r + \rho_{\alpha r}\sigma_{s\alpha r} a'_r\, sin\alpha - \rho_{\beta r}\sigma_{s\beta r} b'_r\, cos\beta + \sigma_{cr}\, sin\theta_r = 0 \tag{8}$$

Being the four equation (5),(6),(7),(8), linearly correlated, σ_{cr} can be derived from both equations (7) and (8) as:

$$\sigma_{cr} = \sigma_{xr} - \tau_{xyr}\, tan\theta_r - \rho_{\alpha r}\sigma_{s\alpha r}\, cos(\theta_r - \alpha)\frac{cos\alpha}{cos\theta_r} + \rho_{\beta r}\sigma_{s\beta r}\, sin(\theta_r - \beta)\frac{sin\beta}{cos\theta_r} \tag{9}$$

or $$\sigma_{cr} = \sigma_{yr} - \frac{\tau_{xyr}}{tan\theta_r} - \rho_{\alpha r}\sigma_{s\alpha r}\, cos(\theta_r - \alpha)\frac{sin\alpha}{sin\theta_r} - \rho_{\beta r}\sigma_{s\beta r}\, sin(\theta_r - \beta)\frac{cos\beta}{sin\theta_r} \tag{10}$$

In the previous equations θ_r should be included in the same quadrant of θ_{er} (angle on x axis of principal tension direction at cracking) and the solutions with nil denominator in equations (5) and (6) correspond to cases in which, in disagreement with the code provisions, only one order of reinforcement is available ($cos(\alpha - \beta) = 0$) or the equilibrium is not possible ($sin(\theta_r - \alpha) = 0$ and $cos(\theta_r - \beta) = 0$).

Design stress range for materials may be expressed for reinforcement as:

$$-1 \leq \sigma_{skr} \big/ f_{yd} \leq 1 \quad (k=\alpha,\beta\ ;\ r=s,i) \tag{11}$$

For concrete stress field the safety criterion proposed in [5] may be adopted:

$$\sigma_{cr} \leq f_{cd2}\left(1-0.032\left|\Delta\theta_r\right|\right) \tag{12}$$

and if no order of reinforcement results to be yelded:

$$\sigma_{cr} \leq f_{cd2}\left(0.85\frac{f_{cd}}{f_{cd2}}-\frac{\sigma_s}{f_{yd}}\left(0.85\frac{f_{cd}}{f_{cd2}}-1\right)\right) \tag{13}$$

where $\Delta\theta_r$ is the deviation between the angle of tensile principal stress at first cracking and plastic compression field in concrete, f_{cd2} is given by (14) in agreement to [8], and σ_s is the highest of the two stresses in each reinforcement layer. (fig.9 and 10)

$$f_{cd2} = 0.6\left(1-\frac{f_{ck}}{250}\right)f_{cd} \tag{14}$$

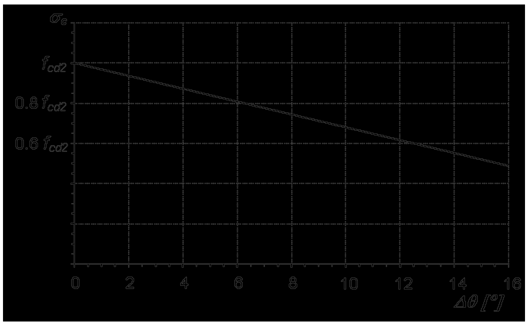

Figure 9 : Safety criterion for concrete (yelded reinforcement)

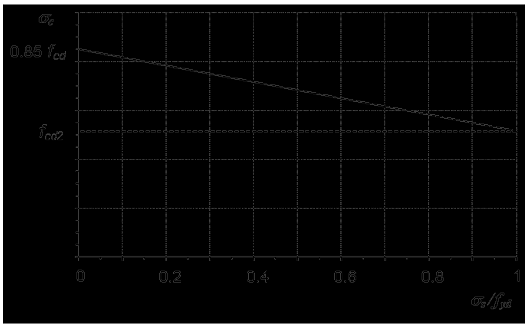

Figure 10 : Safety criterion for concrete (not yelded reinforcement)

The analysis of the shell element may be transferred in a non-dimensional field putting:

$$\xi_r = t_r / t \quad (r=s,c,i)$$
$$c_r^* = c_r / t \quad (r=s,i)$$
$$z_r^* = z_r / t \quad (r=s,i)$$

$$s_j = n_j \big/ (t \cdot f_{cd}) \quad (j=x,y)$$
$$t_{xy} = n_{xy} \big/ (t \cdot f_{cd})$$
$$\mu_j = m_j \big/ (t^2 \cdot f_{cd}) \quad (j=x,y,xy)$$
$$\eta_j = t_j \big/ (t \cdot f_{cd}) \quad (j=x,y,O) \quad \text{(out of the plane shear)}$$

$$s_{jr} = n_j \big/ (t_r \cdot f_{cd}) \quad (j=x,y\ ;\ r=s,i)$$
$$t_{xyr} = n_{xy} \big/ (t_r \cdot f_{cd}) \quad (r=s,i)$$
$$\lambda_{kr} = \sigma_{skr} \big/ f_{yd} \quad (k=\alpha,\beta\ ;\ r=s,i)$$
$$\nu_r = \sigma_{cr} \big/ f_{cd} \quad (r=s,i)$$
$$\omega_{kr} = A_{skr}\, f_{yd} \big/ (t_r \cdot \Delta_{kr} \cdot f_{cd}) \quad (k=\alpha,\beta\ ;\ r=s,i)$$

With the above assumption, equations (2) become:

$$Top\quad\begin{cases} s_{xs} = \dfrac{s_x\left(1-\xi_i\right)+2\mu_x}{\xi_s\left(2-\xi_s-\xi_i\right)} + \dfrac{1}{2\xi_s}\left(+\dfrac{\eta_x^2}{\eta_0}\cot\theta_t\right) \\ s_{ys} = \dfrac{s_y\left(1-\xi_i\right)+2\mu_y}{\xi_s\left(2-\xi_s-\xi_i\right)} + \dfrac{1}{2\xi_s}\left(+\dfrac{\eta_y^2}{\eta_0}\cot\theta_t\right) \\ t_{xys} = \dfrac{t_{xy}\left(1-\xi_i\right)+2\mu_{xy}}{\xi_s\left(2-\xi_s-\xi_i\right)} + \dfrac{1}{2\xi_s}\left(+\dfrac{\eta_y^2}{\eta_0}\cot\theta_t\right) \end{cases}$$

$$Bottom\quad\begin{cases} s_{xi} = \dfrac{s_x\left(1-\xi_s\right)+2\mu_x}{\xi_s\left(2-\xi_s-\xi_i\right)} + \dfrac{1}{2\xi_i}\left(+\dfrac{\eta_x^2}{\eta_0}\cot\theta_t\right) \\ s_{yi} = \dfrac{s_y\left(1-\xi_s\right)+2\mu_y}{\xi_s\left(2-\xi_s-\xi_i\right)} + \dfrac{1}{2\xi_i}\left(+\dfrac{\eta_y^2}{\eta_0}\cot\theta_t\right) \\ t_{xyi} = \dfrac{t_{xy}\left(1-\xi_s\right)+2\mu_{xy}}{\xi_s\left(2-\xi_s-\xi_i\right)} + \dfrac{1}{2\xi_i}\left(+\dfrac{\eta_y^2}{\eta_0}\cot\theta_t\right) \end{cases} \qquad (15)$$

and equations (5),(6),(9),(10) turn to:

$$\lambda_{\alpha r}\omega_{\alpha r} = \frac{1}{sen(\theta_r-\alpha)cos(\alpha-\beta)}\left[s_{xr}sen\theta_r\cos\beta - s_{yr}\cos\theta_r sen\beta + t_{xyr}\cos(\theta_r+\beta)\right] \qquad (16)$$

$$\lambda_{\beta r}\omega_{\beta r} = \frac{1}{cos(\theta_r-\beta)cos(\alpha-\beta)}\left[s_{xr}sen\theta_r sen\alpha + s_{yr}\cos\theta_r\cos\alpha + t_{xyr}sen(\theta_r+\alpha)\right] \qquad (17)$$

$$v_r = s_{xr} - t_{xyr}\tan\theta_r - \lambda_{\alpha r}\omega_{\alpha r}\cos(\theta_r-\alpha)\frac{\cos\alpha}{\cos\theta_r} + \lambda_{\beta r}\omega_{\beta r}sen(\theta_r-\beta)\frac{sen\beta}{\cos\theta_r} \qquad (18)$$

$$v_r = s_{yr} - t_{xyr}/\tan\theta_r - \lambda_{\alpha r}\omega_{\alpha r}\cos(\theta_r-\alpha)\frac{sen\alpha}{sen\theta_r} - \lambda_{\beta r}\omega_{\beta r}sen(\theta_r-\beta)\frac{\cos\beta}{sen\theta_r} \qquad (19)$$

3. Use of genetic algorithms

Genocop III algorithm has been used for the optimization of reinforcement design and concrete verification, because of its performance in optimizing non linearly restrained problems.

This algorithm works on two different parallel and interacting populations: a research population respecting linear restraints and a reference population respecting the whole restraints.

The considered variables are reinforcement areas A_{skr} *($k = \alpha, \beta$; $r = s,i$)* and, for every combination of internal actions, the non-dimensional parameters ξ_s, ξ_i , θ_s , θ_i. The objective of the process is the minimization of the global reinforcement amount ($\Sigma A_{skr} = min$). Restraint conditions are expressed, for every combination of internal actions, by equations (11),(12),(13) and the variable linear restraints are:

- Maximum and minimum reinforcement ratio: $\rho_{min}t/4 \le A_{skr} \le t\rho_{max}/4$
- Layer thickness domain: $2c_r^* \le \xi_r \le 1 \quad (r = s,i)$
- θ_r angle domain: $0 \le \theta_r \le \pi \quad (r = s,i)$
- Global thickness amount: $\xi_s + \xi_i - 1 \le 0$

The extensive use of this algorithm demonstrated that, even changing substantially the input parameters necessary to start the optimization, the standard deviation of the results is very limited, provided that the numerical evaluation number is large enough. That means that the algorithm is able to find in any case correct results, even if the starting point is very far from the actual solution.

4. Conclusions

The methodological approach to design of r.c. and p.c. shell elements, based on a lower bound solution, opens the way to a correct evaluation of resisting contribution of skew reinforcement layers. The implementation of design equations governing the problem in non dimensional terms means the use of a computer program that may be interfaced with a classical structural analysis one, so that the process of analysis and design may be correctly integrated.

Using genetic algorithms in the solution leads to an optimization of global reinforcement amount, verifying, at the same time, the resistance of concrete layers subjected to a compressive stress field, in agreement with the more recent proposals included in references [3],[4],[5].

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