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SAFETY FORMAT FOR THE NONLINEAR ANALYSIS OF CONCRETE STRUCTURES

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ABSTRACT

Nonlinear analysis of concrete structures and, more specifically, of concrete bridges is a powerful tool to understand the structural behaviour in both the serviceability and ultimate limit states, and to verify the actual safety margins under unusual live loads, as well as under severe indirect actions (like settlements) and accidental combinations of actions. Furthermore nonlinear analysis may become a necessity for the evaluation of the safety level of existing bridges, should the live loads be increased because of new traffic needs and more complex action combinations be considered.

Within this context, a proposal for a safety format concerning the non linear analysis of concrete structures consisting of linear, 2D and 3D elements is presented. Some examples are discussed too, and the necessity to take account model uncertainties, (where relevant) are emphasized.

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The examples refer to linear elements (bridge deck and pier) and two dimensional elements (continuous deep beam) and clearly put in evidence the cases in which model uncertainties should be explicitly taken into account

1 INTRODUCTION

Nonlinear analysis in concrete structures and, specifically, in particular, in concrete bridges is a powerful tool to understand the structural behaviour, both in serviceability and ultimate conditions, and to verify the actual safety margins under unusual live loads, severe indirect actions (like settlements), accidental combinations of actions. Furthermore the nonlinear analysis may become a necessary tool for the evaluation of the safety level of existing bridges to be subjected to updated live loads.

The fundamental parameters of non linear analysis are clearly established within EN 1990 [1] and EN 1992-1-1 [2], and give a guidance to define completely the safety format that should be used, in agreement with the established issues of the research activity in this field performed during the last 50 years.

In the following, firstly the relevant clauses of EN 1990 and EN 1992-1-1 will be referred to, and then the safety format proposed in EN 1992-2 will be discussed and carried out; lastly, some examples will be developed in detail.

2 CLAUSES OF EN 1990 RELEVANT FOR SAFETY FORMAT IN NONLINEAR ANALYSIS

The first relevant clause of EN 1990 is 6.3.2 “Design values of the effects of the actions”; the expression (1) states

$$E_d = \gamma_{Sd} E\{\gamma_{f,i} F_{rep,i}; a_d\} \qquad i \geq 1 \tag{1}$$

in which:

E_d is the design value of the effect of the actions

E is the effect of the actions

$\gamma_{f,i}$ is the partial safety factor for the action “*i*”

$F_{rep,i}$ is the representative value of the action “*i*”

a_d are the design values of geometrical data

γ_{Sd} is a partial safety factor taking account of uncertainties:

- in modelling the effects of actions;
- in some cases, in modelling the actions.

In most cases expression (1) can be simplified in (1a)

$$E_d = E(\gamma_{F,i} F_{rep,i}; a_d) \qquad i \geq 1 \tag{1a}$$

with

$$\gamma_{F,i} = \gamma_{Sd} \gamma_{f,i}$$

where $\gamma_{F,i}$ is the global safety factor for the actions (1b)

Expression (1b) works in particular within the linear field, in which of course the distinction between $\gamma_{F,i}$ and $\gamma_{f,i}$ has no effect.

The second clause of EN 1990 is 6.3.5 "Design resistance"; the expression (2) states

$$R_d = \frac{1}{\gamma_{Rd}} R\{X_{d,i}; a_d\} = \frac{1}{\gamma_{Rd}} R\left\{\eta_i \frac{X_{k,i}}{\gamma_{m,i}}; a_d\right\} \qquad i \geq 1 \tag{2}$$

where:

R_d is the design value of the resistance

R is the resistance

$X_{d,i}$ is the design value of the "*i*" material property

$X_{k,i}$ is the characteristic value of the "*i*" material property

η_i is a conversion factor

$\gamma_{m,i}$ is the partial safety factor fot the "*i*" material property

and γ_{Rd} is a partial factor covering the uncertainties in the resistance model, plus the geometric deviations, if these are not modelled explicitly.

As a simplification expression (2) may be transformed in (2a)

$$R_d = R\left\{\eta_i \frac{X_{k,i}}{\gamma_{M,i}}; a_d\right\} \; i \geq 1 \tag{2a}$$

with

$$\gamma_{M,i} = \gamma_{Rd} \gamma_{m,i} \tag{2b}$$

where $\gamma_{M,i}$ is the global safety factor for the "*i*" material property (1b)

From the above mentioned expressions (1) and (2) it results clearly the opportunity to put in evidence the model uncertainties, both on actions and resistances, in the case in which the relationship between the actions and their effect is not linear (nonlinear analysis).

Finally chapter 6.4 of E.N. 1990 "Ultimate limit states", with reference to the ultimate limit state (USL) of internal failure or excessive deformation (STR), which is relevant for nonlinear analysis, gives some further clear guidance criteria; 6.4.3.2 establishes the general format of the effects of actions as:

$$E_d = \gamma_{Sd} E\{\gamma_{g,j} G_{k,j}; \gamma_p P; \gamma_{q,1} Q_{k,1}; \gamma_{q,i} \Psi_{0,i} Q_{k,i}\} \qquad j \geq 1 \; ; \; i \geq 1 \tag{3a}$$

or

$$E_d = E\{\gamma_{G,j} G_{k,j};\gamma_P P;\gamma_{Q,1} Q_{k,1};\gamma_{Q,i} \Psi_{0,i} Q_{k,i}\} \qquad j \geq 1 \; ; \; i \geq 1 \qquad (3b)$$

where:

$\gamma_{g,j}$ is the partial factor for permanent action *"j"*

$G_{k,j}$ is the characteristic value of action *"j"*

γ_p is the partial factor for prestressing actions

P is the relevant representative value of prestressing actions

$\gamma_{q,1(i)}$ is the partial factor for variable action *"1"* (*"i"*)

$Q_{k,1(i)}$ is the characteristic value of variable action *"1"* (*"i"*)

$\Psi_{0,i}$ is the combination factor value of the variable action *"i"*

$\gamma_{G,j}$ is the global factor for permanent action *"i"*

γ_P is the global factor for prestressing actions

$\gamma_{Q,1(i)}$ is the global factor for variable action *"1"* (*"i"*)

In addition 6.4.2 (3) P states that: "Where considering a limit state of rupture or excessive deformation of a section, member or connection (STR and/or GEO) it shall be verified that

$$E_d \leq R_d \qquad (4)$$

where GEO stands for excessive deformation or failure of the ground.

It is then clear that:

- the safety verification should be performed within the internal actions and corresponding resistance domain (4);
- expressions (1), (2), (3a) give the possibility to take account of model uncertainties both on actions and resistances.

Of course the choice to introduce the model uncertainties in the safety format for nonlinear analysis is based on the engineering judgment, on the basis of the problem in question; the same applies to the level to which model uncertainties should be considered, that is to the ratio γ_f/γ_{Sd} or γ_m/γ_{Rd}, but respecting (1b) and (2b).

3 CLAUSES OF EN 1992-1-1 RELEVANT FOR SAFETY FORMAT IN NONLINEAR ANALYSIS

The first relevant clause of EN 1992-1-1 is 3.1.5 "Stress-Strain relation for non-linear analysis" in which expression (3.14) (Sargin expression) is suggested to describe concrete σ-ε relationship; this relationship is obviously different by those suggested for the design of cross-sections in 3.1.7 (parabola-rectangle expression).

The second important clause for definition of safety format in nonlinear analysis is 5.7, where:

- 5.7 (1) states the necessity that ”... an adequate non linear behaviour for materials is assumed”, that is the use of expression (3.14).
- 5.7 (2) states that “ the ability of local critical sections to withstand any inelastic deformations implied by the analysis should be checked, taking appropriate account of uncertainties “, that is model uncertainties should be taken into account, explicitly in model definition or implicitly by use of γ_{Rd} and or γ_{Sd} (see EN 1990).
- 5.7 (3) states that ”... a monotonic increase of the intensity of the actions may be assumed”, that is all the actions should increase proportionally from the serviceability to the ultimate condition, in agreement with the combination values defined by γ and ϕ coefficients.
- 5.7 (4) P states that “the use of material characteristics, which represent the stiffness in a realistic way but take account of the uncertainties of failure, shall be used when using nonlinear analysis”, that is, again, a strong indication to the use of expression (3.14) and to take account of model uncertainties.

4 DEFINITION OF A SAFETY FORMAT FOR NONLINEAR ANALYSIS IN AGREEMENT WITH THE INDICATIONS MENTIONED IN POINT 2 AND 3

A generalized approach for safety format in nonlinear analysis recently proposed [3] is based on the consideration that the scattering of material properties and direct/indirect actions is known and valuable by means of their stochastic distribution functions. Then, in statically indetermined structures, only the sensivity of the overall structural behaviour to the scattering of those variables remains to be investigated.

If we remain within the field of semiprobabilistic approach, a new safety coefficient related to the structural strength should then be defined, γ_{Gl}, which may be interpreted as a structural strength reserve due to redundancy. For the assigned action distribution, this safety factor should cover the probability that, the strength values all along the structure reach the design values.

In practice, with this approach, nonlinear analysis with strength mean-values yields the ultimate load q_{ud}; when, in the incremental process, the ultimate strains in steel or in concrete are reached in a region whose failure determines the attainment of the peak load in the structure, the γ_{Gl} safety margin is applied to the structural strength of this region.

In a first step it has been proposed to use two different values for γ_{Gl}; considering that $f_{cm} \cong 1.1 f_{ck}$ and $f_{ym} \cong 1.1 f_{yk}$ it results that γ_{Gl} assumes the value $\gamma_{Gl} = 1.1 \cdot 1.5 \cong 1.7$ for structures where the concrete fails first and $\gamma_{Gl} = 1.1 \cdot 1.15 \cong 1.3$ where the reinforcement fails first. However it has been remarked that such a higher level of γ_{Gl} in case of concrete failure turns out to be a strong penalization for structures subjected to second-order effects, due to the over-proportional relationship between internal and external actions.

Moreover in several cases in which the ULS is reached practically in both materials for the same action level, the adoption of two different γ_{Gl} values can give rise to a discontinuity in the definition of structural safety.

As an answer to these remarks, it has been proposed to modify the $\sigma - \varepsilon$ relationship for concrete, considering that, according to an extensive experimental research [4], the ratio between the 5% fractile of the actual strength in a structural member and the characteristic strength is 0.85, that is:

$$f_{c,structure,0,05} / f_{ck} = 0.85 \tag{5}$$

Then, if one accepts for concrete a reference strength of $0.85 f_{ck}$, the corresponding γ_{Gl} coefficient assumes the value $\gamma_{\mathrm{Gl}} = 0.85 \cdot 1.5 \cong 1.3$. As a consequence a common value $\gamma_{\mathrm{Gl}} = 1.3$ for both material failures can be assumed for the structural strength.

Moreover this new safety format has been initially formulated within the action domain, as;

$$\gamma_G G + \gamma_Q Q \leq \frac{q_{ud}}{\gamma_{Gl}} \tag{6}$$

More recently three main remarks have been made on this proposal [5]:

- the so-defined global safety factor applied in the actions domain is not able to distinguish the different structural behaviour in regions in which the limit strains for materials are reached (linear, over proportional, under proportional), because it is purely applied to the maximum value of direct/indirect actions reached in the analysis, with no consideration for the internal-action path;
- the proposed safety format, being applied in the actions domain, is not consistent with the semiprobabilistic approach, in which acting external and resisting internal actions are compared;
- this format is not able to take account of model uncertainties on both acting and resisting sides, in spite of their fundamental importance in nonlinear processes, in which only the scattering of material properties is generally taken into account.

To overcome these remarks, maintaining the approach of a global safety coefficient, the safety format should be transferred in the external and internal actions domain, that is:

$$E(\gamma_G G + \gamma_Q Q) \leq R\left(\frac{q_u}{\gamma_{Gl}}\right) \tag{7}$$

where q_u is the maximum level of the direct/indirect actions reached in nonlinear analysis, performed with the materials strength $0.85 f_{ck}$ and f_{ym}.

In such a manner one can answer to the previous remarks since:

- the safety format expression (7) is consistent with the semiprobabilistic approach;
- the comparison between E and R automatically takes account of the structural behaviour;

- model uncertainties on both action and resisting sides may be explicitly taken into account by splitting the $\gamma_G, \gamma_Q, \gamma_{Gl}$ coefficients.

In agreement with the last point the inequality (7) may be modified into:

$$\gamma_{Rd} E(\gamma_G G + \gamma_Q Q) \leq R\left(\frac{q_u}{\gamma_{gl}}\right) \tag{8}$$

$$\gamma_{Sd}\gamma_{Rd} E(\gamma_g G + \gamma_q Q) \leq R\left(\frac{q_u}{\gamma_{gl}}\right) \tag{9}$$

The inequalities (8) and (9) should be alternatively used as safety format for nonlinear analysis, according to the importance assumed by model uncertainties on the action side within the specific design.

The application of the safety format described in (8) and (9) requires further comments according to whether the safety verification is performed within the scalar or vectorial field. In Fig. 1 the application of the proposed safety format ($\gamma_{Sd} = 1, \gamma_{Rd} \neq 1$) is shown for three different internal-action paths in the scalar field: over proportional, linear, under proportional; the corresponding final point of the procedure (G, G', G'') defines the maximum value of action combination $(\gamma_G G + \gamma_Q Q)$ compatible with the required safety level.

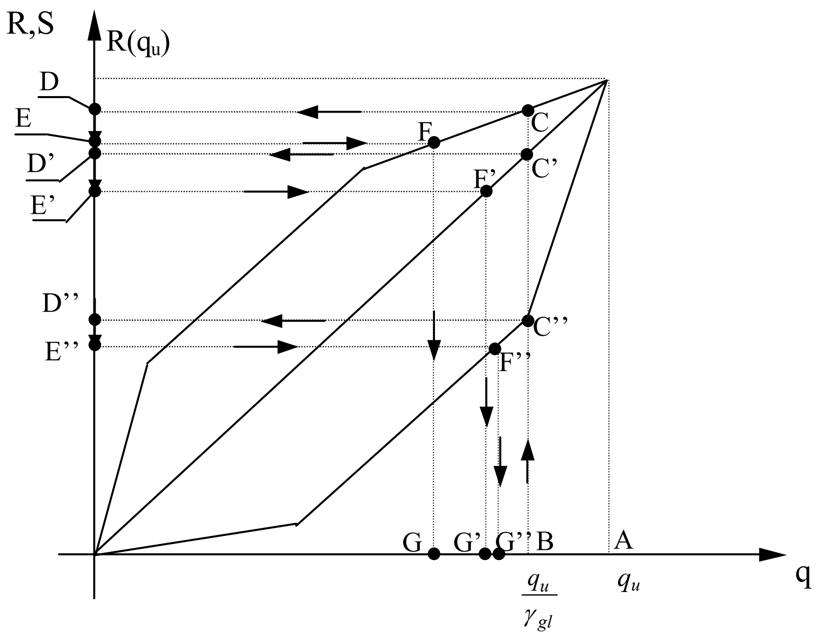

Fig. 1 – Application of the proposed safety format for scalar problems

From the figure it is also clear that the effect of taking into account model uncertainties in the resistance side is stronger in the over proportional than in the under proportional behaviour, because the first one receives a greater benefit by steel plasticization whereas the second one is penalized by the redistributions and/or by the intervention of second order effects.

In case of vectorial combination of internal actions, like N, M_x, M_y or n_x, n_y, n_{xy}, the safety format application is shown in Fig. 2 and Fig. 3 (considering only N, M_x combination for the sake of simplicity) for the case of under proportionality and over proportionality in M respectively.

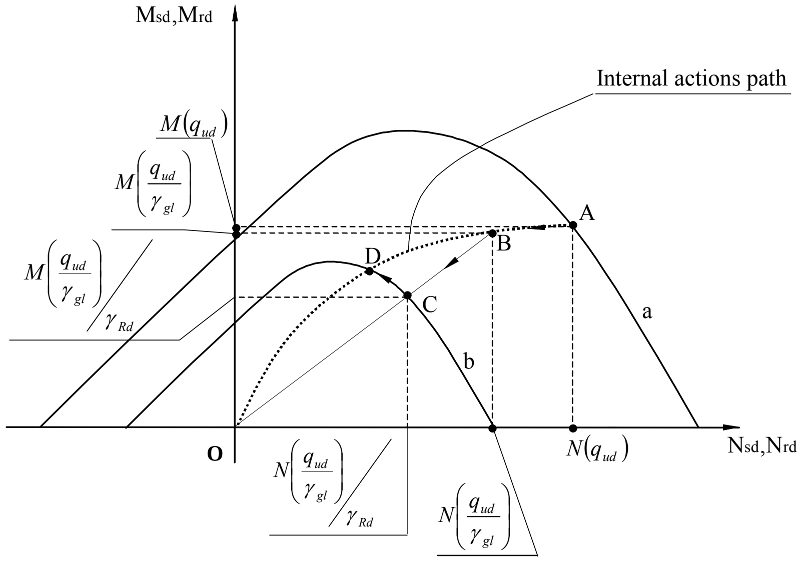

Fig. 2 – Application of the proposed safety format for vectorial (M,N) under proportional behaviour in M

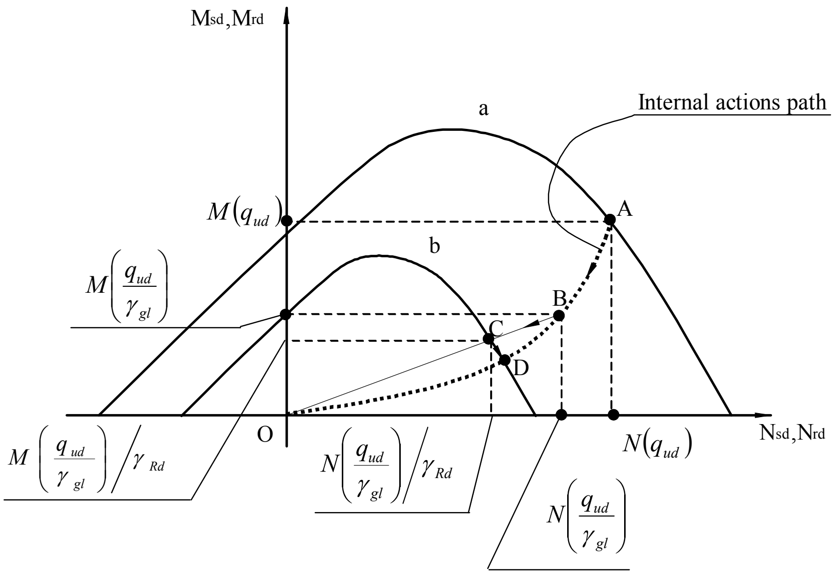

Fig. 3 – Application of the proposed safety format for vectorial (M,N) over proportional behaviour in M

In figures 2 and 3, point A is the final step in the analysis and the curve “a” represents the safety domain N,M obtained with the same material strength used for the analysis. By applying γ_{gl}, one shifts from point A to B along the internal-action path; at this point the linearization should be performed vectorially, by reducing the vector $\overline{OB}$ by the ratio γ_{Rd}. Now point C is reached, whose distance from the safety domain is a measure of the safety level required. In general, point C does not belong to the internal-action path. Then a point D, with the same safety level as C, can be identified at the intersection of the curve “b”, homothetic domain to “a” passing by C, with the internal action path.

The final verification requires that the point representing the design combination $M(\gamma_G G + \gamma_Q Q)$; $N(\gamma_G G + \gamma_Q Q)$ remains along the internal-action path inside the homothetic safety domain “b”.

The same procedure applies tridimensionally in case of the combination of N, M_x, M_y or n_x, n_y, n_{xy}. It is then clear that in the case of a vectorial combination of actions, the safety format requires the knowledge of the safety domain related to the same strength distribution used in the analysis.

5 SAFETY FORMAT FOR NONLINEAR ANALYSIS PROPOSED EN 1992-2

In agreement with the issues of point 4, the following safety format has been proposed for the EN 1992-2:

- the reinforcing steel is described by means of its mean properties, using the idealized stress-strain diagram of Fig. 3.8 of EN 1992-1-1, but replacing f_{yk} and kf_{yk} respectively with 1.1 $f_{yk} = f_{ym}$ and $1.1kf_{yk} = kf_{ym}$;
- the prestressing steel is described by means of its mean properties, using the idealized stress-strain diagram of Fig. 3.10 (curve A) of EN 1992-1-1, but replacing f_{pk} with $1.1f_{pk} = f_{pm}$;
- the concrete is described by means of expression (3.14) of EN 1992-1-1, but replacing f_{cm} (also in the definition of k value) with $\gamma_{cf}f_{ck}$ where $\gamma_{cf} = 1.1\gamma_s/\gamma_c$; this implies that $\gamma_{cf} = 1.1\times1.15/1.5 = 0.843$, in line with the results presented in [4].

The global safety coefficient γ_{Gl} to be applied to the actions assumes the value

$\gamma_{Gl} = 1.15\times1.1 = 1.27$ when the limit deformation is reached in steel

$\gamma_{Gl} = 1.5\times0.843 = 1.27$ when the limit deformation is reached in concrete

Then, whichever may be the critical material, the same global safety coefficient γ_{Gl} should be applied to the ultimate load q_{ud}.

The expressions describing the safety format are then:

$$\gamma_{Rd} E(\gamma_G G + \gamma_Q Q) \le R\left(\frac{q_{ud}}{\gamma_{gl}}\right) \tag{10a}$$

or

$$\gamma_{Rd}\gamma_{Sd} E(\gamma_g G + \gamma_q Q) \le R\left(\frac{q_{ud}}{\gamma_{gl}}\right) \tag{10b}$$

The maximum recommended values for γ_{Rd} and γ_{Sd} are γ_{Rd}=1.06 and γ_{Sd}=1.15; as a consequence γ_{gl}=1.20 because $\gamma_{gl}\times\gamma_{Rd}=1.20\times1.06=\gamma_{Gl}=1.27$.

The maximum value for γ_{Sd} can be found in the literature in several CEB Bulletins (see for instance [6] and [7]).

Expressions (10a) and (10b) may be simplified should the designer decide that model uncertainties be directly modelled or that the structure be not sensitive to model uncertainties; in this case the safety format becomes:

$$E(\gamma_G G + \gamma_Q Q) \leq R\left(\frac{q_{ud}}{\gamma_{Gl}}\right) \tag{11}$$

Expression (11), when applicable, gives some practical advantages in the nonlinear procedures, because the linearization is not necessary; in fact, both in scalar and vectorial combinations of internal actions, the safety verification may be directly performed between the scalar or vectorial components of the internal acting and resisting actions. For instance:

- for a beam $M_{Ed} \leq M_{Rd}(q_{ud}/\gamma_{Gl})$ (12)
- for a column $M_{Ed} \leq M_{Rd}(q_{ud}/\gamma_{Gl})$ and $N_{Ed} \leq N_{Rd}(q_{ud}/\gamma_{Gl})$ (13)
- for a plate $n_{Edx} \leq n_{Rdx}(q_{ud}/\gamma_{Gl})$ and $n_{Edy} \leq n_{Rdy}(q_{ud}/\gamma_{Gl})$ and $n_{Edxy} \leq n_{Rdxy}(q_{ud}/\gamma_{Gl})$ (14)

where the resisting internal actions are evaluated along the internal-action path at the level q_{ud}/γ_{Gl}.

6 APPLICATION TO A TWO-SPAN REINFORCED-CONCRETE BRIDGE

For the application of the proposed safety format is considered a two-span continuous bridge built with the advance-shoring system. The first span and 25% of the second span are cast first; then the remaining part of the second span (75%) is cast. The dead load is applied to the first part at t_0 = 28 days and to the second part at t_1 = 90 days (supposed to be coincident with the time of variation of statical scheme). The corresponding value at time t = ∞ of the function ξ is $\xi(28,90,\infty) = 0.51$. Two sets of non linear analyses are then performed: the first one at t_1 (with no redistribution of the internal actions due to creep) and the second one at $t=\infty$ (with full redistribution due to creep).

The bridge spans 20+20m, the deck width is 6.50m, the depth 1.40m; the section has a double tee shape, see fig. 4 (slab thickness of 0.30m, two webs 1.10×0.50m and two cantilever of 1.25m; total width = 1.25+0.50+3.00+0.50+1.25 = 6.50m).

The material parameters are: f_{ck} = 35Mpa, f_{yk} = 500Mpa, ε_{uk} = 75/1000, $k \leq (f_t/f_y)_k$ = 1.15. For the evaluation of dead load the density of concrete is 2500 kg/m^3, and the structure's self weight is 77.5 kN/m. The permanent load is 23.9 kN/m (paving, kerbs and safety barriers) and is supposed to be applied at time t_1 for simplification.

The live load considered in the design agrees with the load model LM1 of EN 1991.2 that is two axle of 300 kN each with a spacing of 1.20m and a uniformly-distributed load of 27 kN/m; only one lane has been considered in the design; in the remaining area, having a global with of 1.15*2 = 2.30m a further uniform distributed load of 5.75 kN/m is applied.

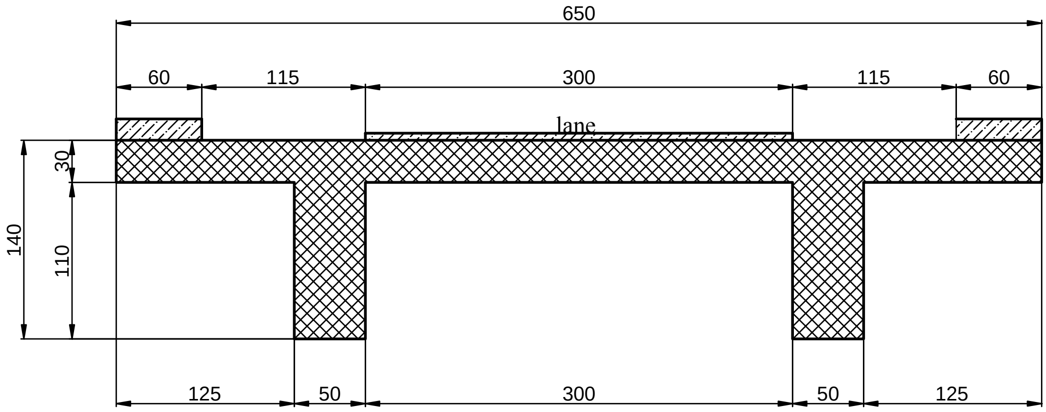

Fig. 4 – Bridge cross-section

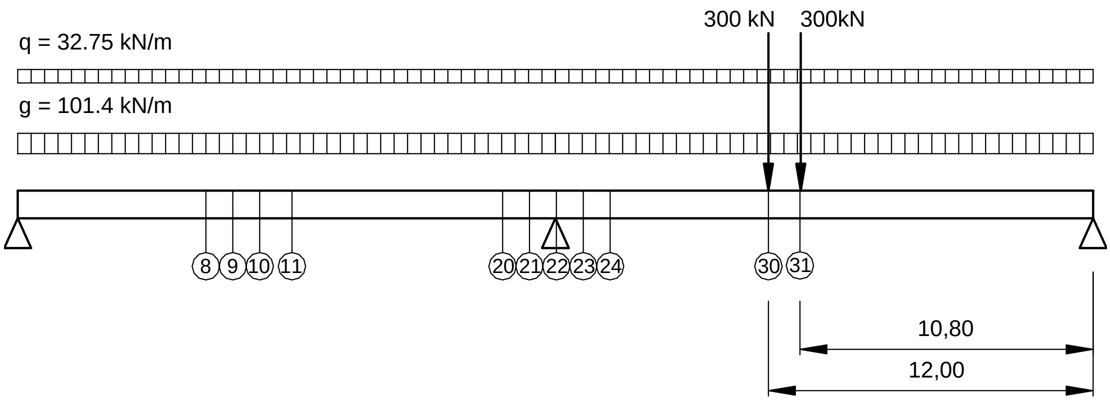

Fig. 5 – Load distribution for the design of the region close to the central support

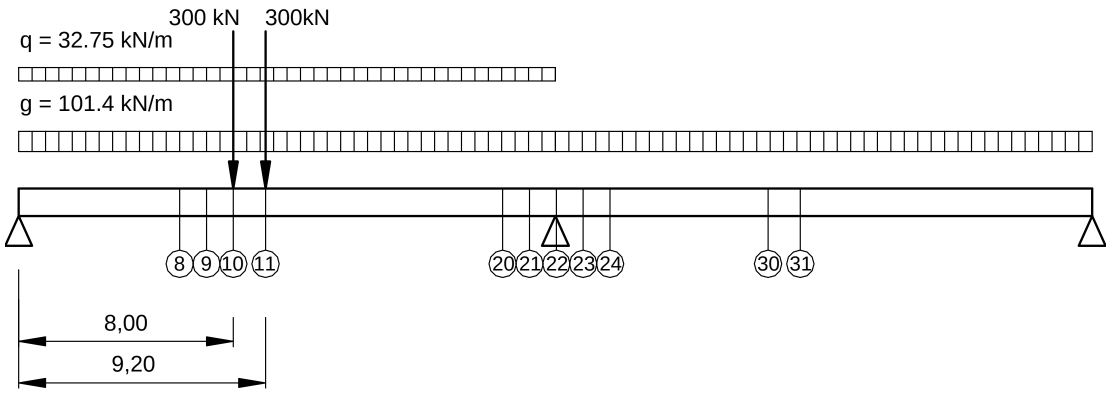

Fig. 6 – Load distribution for the design of the midspan

The preliminary design of the reinforcement based on linear elastic analysis and the following two live-load geometrical distributions are considered:

- for the design of the intermediate region, astride the central support, (fig 5), uniformly distributed loads along both spans and axle loads at a distance of 10.80 and 12.00m from one of the end support;
- for the design of the midspan region (fig.6), uniformly distributed loads along one span and axle loads at a distance of 8.00 and 9.20m from one of the end support;

In Fig. 5 and 6 also the sections considered in the analysis are shown (8÷31).

Table 1 gives the reinforcement coming from the linear-elastic design and used in the non-linear analysis, and figure 7 shows the related arrangement.

Table. 1 – Longitudinal renforcement disposition

Region	from [m]	to [m]	A's [mm^2]	As [mm^2]
A	0	2	4595	9190
B	2	12	4595	18379
C	12	15	18912	18379
D	15	25	18912	9190
C	25	28	18912	18379
B	28	38	4595	18379
A	38	40	4595	9190

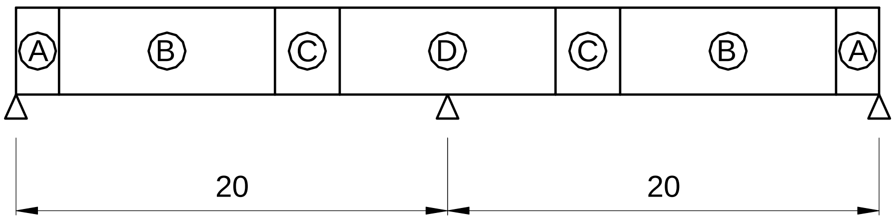

Fig. 7 – Reference regions for the arrangement of the reinforcement

The material parameters are derived by the characteristic ones following the indications of 5.7(106) of EN 1992-2 stage 49-March 2004.

The incremental loading process up to failure is performed in agreement with the following steps:

1. application of the self weight in the different statical schemes corresponding to the construction procedure with $\gamma_G = 1.0$;
2. modification of internal actions due to creep by means of ξ function ($\gamma_G = 1.0$), only for the analysis with $t_1 = t_\infty$;
3. application of the other permanent actions ($\gamma_G = 1.0$) on the final statical scheme;
4. application of the remaining live loads in their relevant position with $\gamma_Q = 1$;

5. starting of the incremental process, by increasing step-by-step all the actions, so that $\gamma_G = 1.4$ and $\gamma_Q = 1.5$ are reached simultaneously in the same step;
6. continuation of the incremental process with the same path up to the attainment of the peak load and the failure of the critical region.

Four different non linear analyses have been performed in four different load cases:

1. maximum negative bending moment at time $t=t_1$(Fig. 5);
2. maximum negative bending moment at time $t=\infty$ (Fig. 5);
3. maximum positive bending moment at time $t=t_1$(Fig. 6);
4. maximum positive bending moment at time $t=\infty$ (Fig. 6);

The critical section governing the analysis results to be in the central support region in each load combination, because of its limited redistribution capacity.

Applying the safety format to the central support section for these four analysis we can obtain the results summarized in Tables 2 and 3, where γ_{Gu} and γ_{Qu} represent the multipliers of the serviceability loads attained by the structure at failure.

Table. 2 – Safety format: γ_{Gl}

Load case (bending moments)	Time	γ_{Gu}	γ_{Qu}	γ_{Gu}/γ_{Gl}	γ_{Qu}/γ_{Gl}	$M(\gamma_{Gl})^*$ [kNm*10^3]	Gain
Maximum negative (X)	t_1	2.03	2.175	1.60	1.71	-12.2	14%
Maximum negative (Y)	∞	2.03	2.175	1.60	1.71	-12.2	14%
Maximum positive (W)	t_1	1.97	2.11	1.55	1.66	-10.4	11%
Maximum positive (Z)	∞	1.92	1.07	1.51	1.62	-10.1	7.8%

Where:

$$^* M(\gamma_{Gl}) = M\left(\frac{\gamma_{Gu}\cdot G + \gamma_{Qu}\cdot Q}{\gamma_{Gl}}\right)$$

Table. 3 – Safety format: γ_{gl}

Load case (bending moments)	Time	γ_{Gu}/γ_{gl}	$M(\gamma_{gl})^*$ [kNm*10^3]	$M(\gamma_{gl})/\gamma_{Rd}$ [kNm*10^3]	$\gamma_G\left(M(\gamma_{gl})/\gamma_{Rd}\right)$	Gain Γ
Maximum negative (X)	t_1	1.69	-12.0	-11.3	1.60	14%
Maximum negative (Y)	∞	1.69	-12.2	-11.5	1.53	7.1%
Maximum positive (W)	t_1	1.64	-9.96	-9.40	1.55	11%
Maximum positive (Z)	∞	1.60	-10.6	-10.0	1.51	7.8%

Where:

$$^* M(\gamma_{gl}) = M\left(\frac{\gamma_{Gu}\cdot G + \gamma_{Qu}\cdot Q}{\gamma_{gl}}\right)$$

where the gain represents the incremental percentage reached by γ values between the non linear and the corresponding linear solution :

$$\Gamma = \frac{\gamma_{Gu} - 1.4}{1.4} = \frac{\gamma_{Qu} - 1.5}{1.5}$$

Figures 8, 9, 10, 11 illustrate the internal-action path for the different load cases (X÷Z) and for several sections. Bold lines refer to the actual section which determines the attainment of the structural peak load, and in all cases the section is number 22 (intermediate support).

Tables 2 and 3 and figures 8-11 show clearly that: the introduction of model uncertainties reduces the gain (Γ) only in one case (Y) corresponding to the load case for maximum negative moment and time $t=\infty$; in fact in such case the redistribution due to creep modifies the internal-action path so that the non linear behaviour begins for a lower γ_G (γ_Q) value with respect to the corresponding ones at $t=t_1$. This phenomenon is related to the increase in the time of negative bending moment due to creep. In fact, from the figures 8÷11 it's clear that the application of the linearization procedure (of γ_{gl} and γ_{Rd}) falls within the linear region of the structural behaviour yet after the application of γ_{gl} , for the case X, and only after the application of γ_{Rd} , for the case Y.

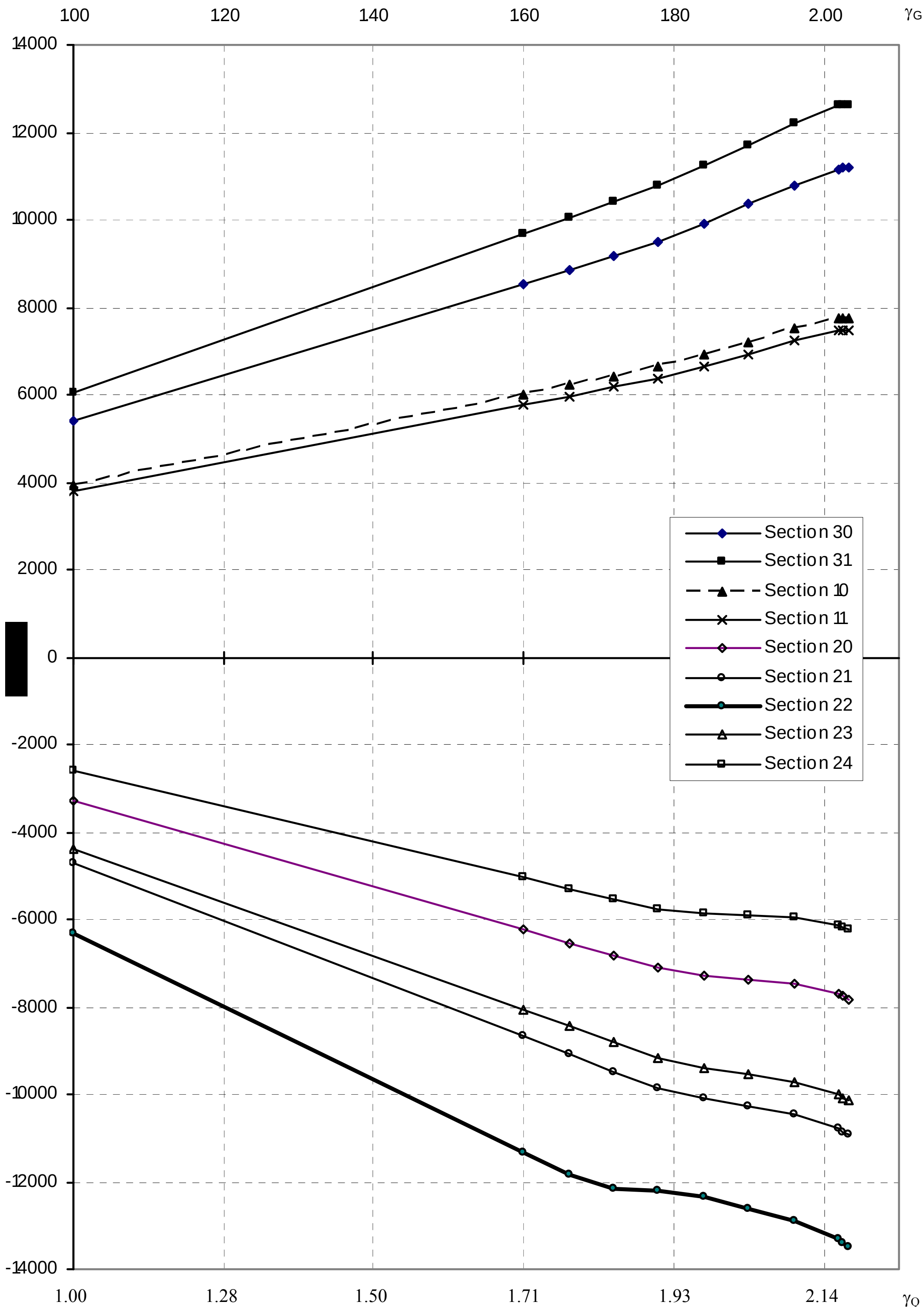

Fig. 8 – Bending moment for load case X (maximum negative $t=t_1$)

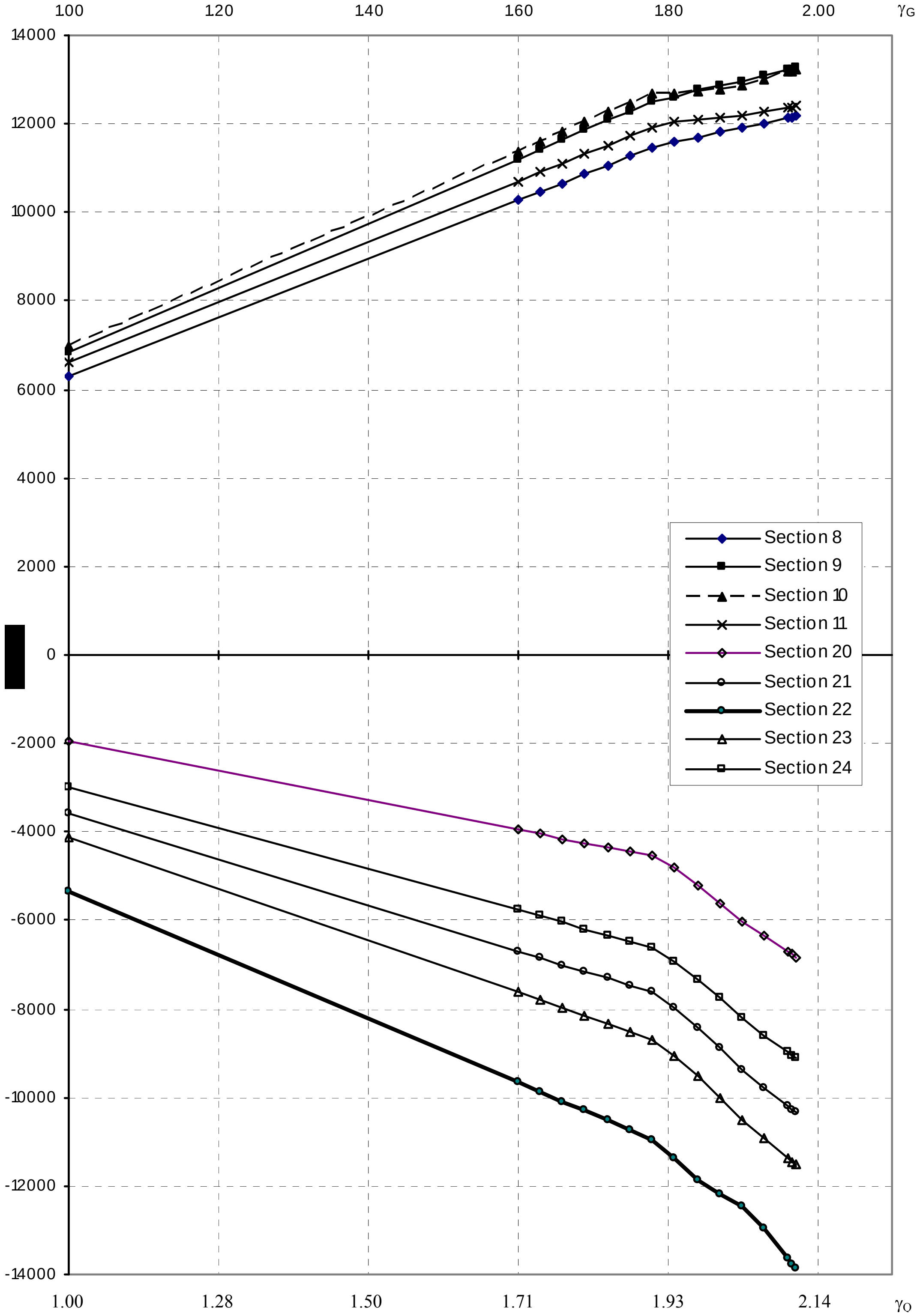

Fig. 9 – Bending moment for load case W (maximum positive $t=t_1$)

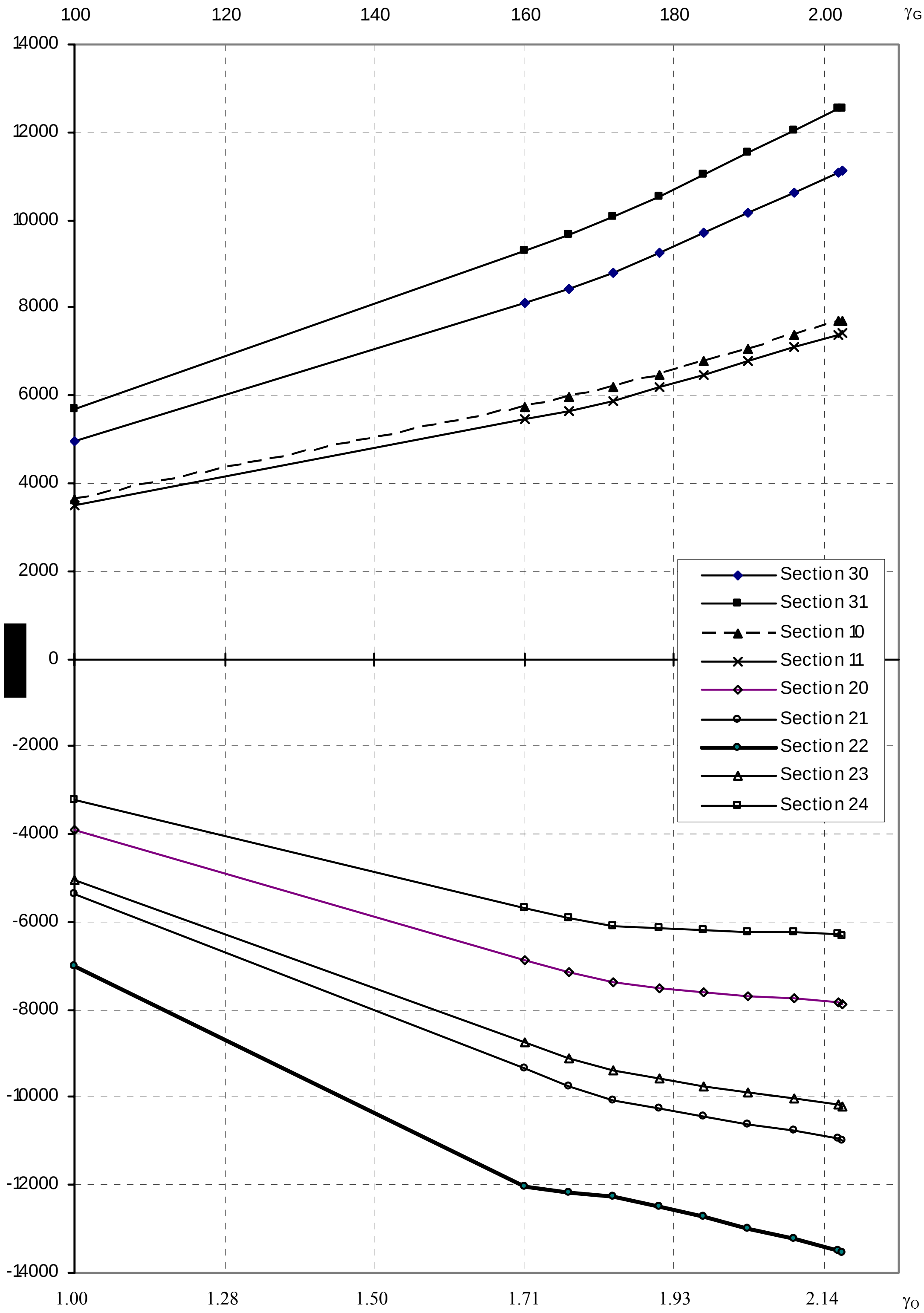

Fig. 10 – Bending moment for load case Y (maximum negative t=∞)

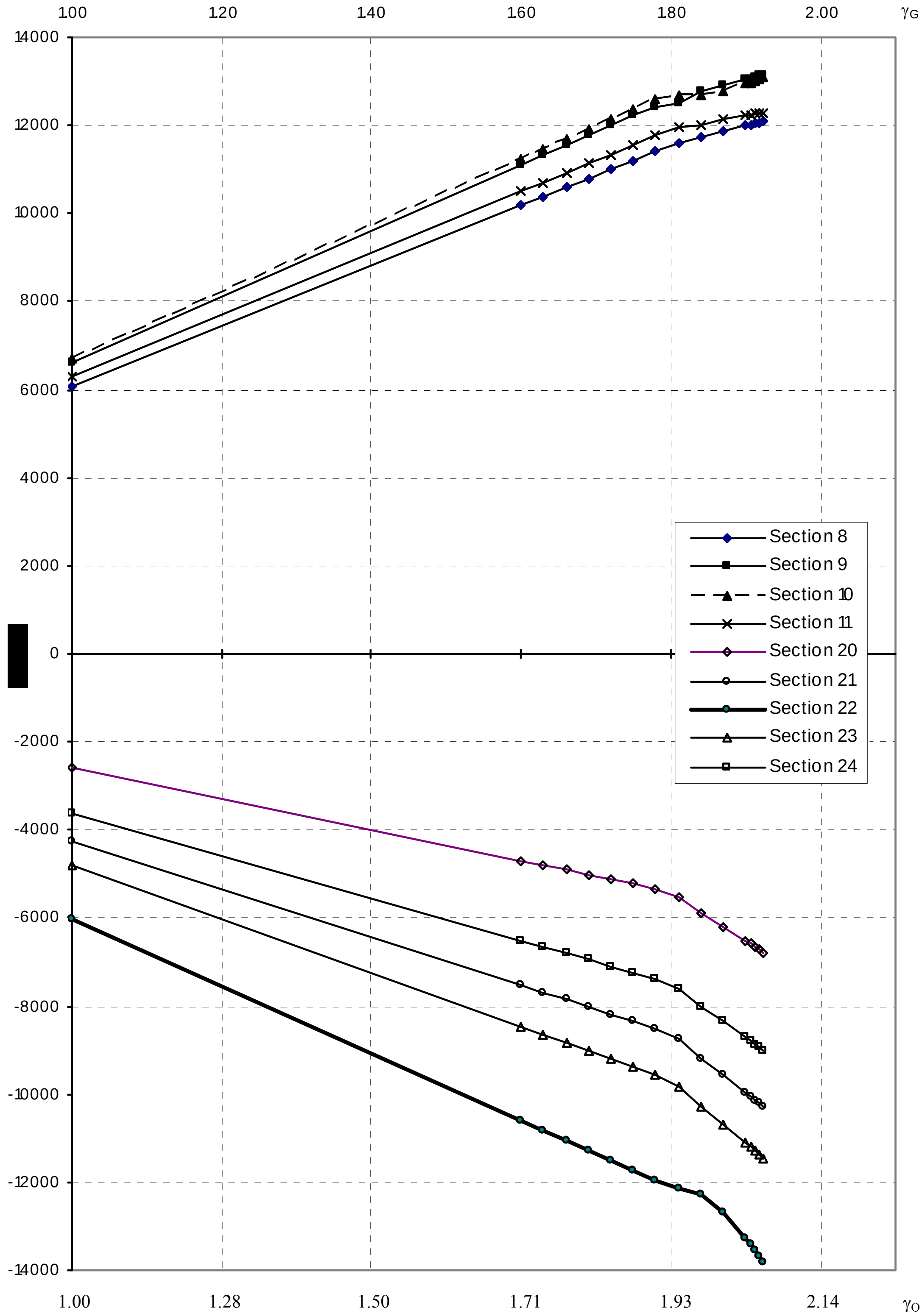

Fig. 11 – Bending moment for load case Z (maximum positive t=∞)

7 APPLICATION TO A SET OF SLENDER PIERS

The proposed approach in now applied to a set of 4 bridge box piers, having different heights (82, 87, 92 and 97 m.). Each pier has a variable section, wall thickness and reinforcement ratio along its height (Fig. 12).

Material properties: $f_{ck} = 29\ \text{MPa}$ and $f_{yk} = 430\ \text{MPa}$.

Cross section: a square box section with variable sides and thickness. Base dimensions: 9.8m x 9.8m; then the parabolic law given in (15), to a height of 63,33 m has been adopted:

$$y = -0.005 \cdot x^2 + 0.0597 \cdot x \tag{15}$$

where y is the narrowing of half transverse dimension and x is the height from the base of the pier (up to 63.33m). From 63,33 m the section is constant 6.0 x 6.0m for 5,10,15 and 20 m respectively for the pier depths of 82, 87,92 and 97 m. On the last 10 meters at the top of the pier a new parabolic shape is assumed following the law given in (16)

$$y = 2{,}50 \cdot 10\text{-}3 \cdot x2 - 5{,}00 \cdot 10\text{-}2 \cdot x \tag{16}$$

where y has the same meaning of (15) and x is the distance from the capital intrados (seee fig. 12). From the base section up to 22.52 m the walls thickness is 1m, then it becomes 0.80 m for a following depth of 30,80 m. The constant section zone and the above parabolic 10.00 m zone are 0.60 m thick.

Longitudinal main reinforcement is arranged in three regions following the walls thickness as summarized in Table 4.

Table. 4 – Longitudinal reinforcement

	from [m]	to [m]	As on internal and external faces
Region A	0	22.52	ϕ 24/ 40
Region B	22.52	53.32	ϕ 22/ 40
Region C	53.32	Top	ϕ 20/ 40

An unforeseen eccentricity of 5/1000 h has been introduced.

The four non linear analysis have been performed starting from the most unfavourable load case in the design of the 82m pier. This load case consists of the actions transmitted by the bridge deck (Table 5), and gives rise to a bending moment in the direction at right angles to the deck axis. These actions were divided into small steps and monotonically increased following the points 1, 4, 5, 6 of paragraph 6. In this example γ_G and γ_Q were put equal to 1.5 (for simplification).

Table. 5 – Characteristic values of the actions at the top of the piers

Axial force	N	-55352 [kN]
Bending moment	M	30923 [kNm]
Horizontal force	H	1661 [kN]

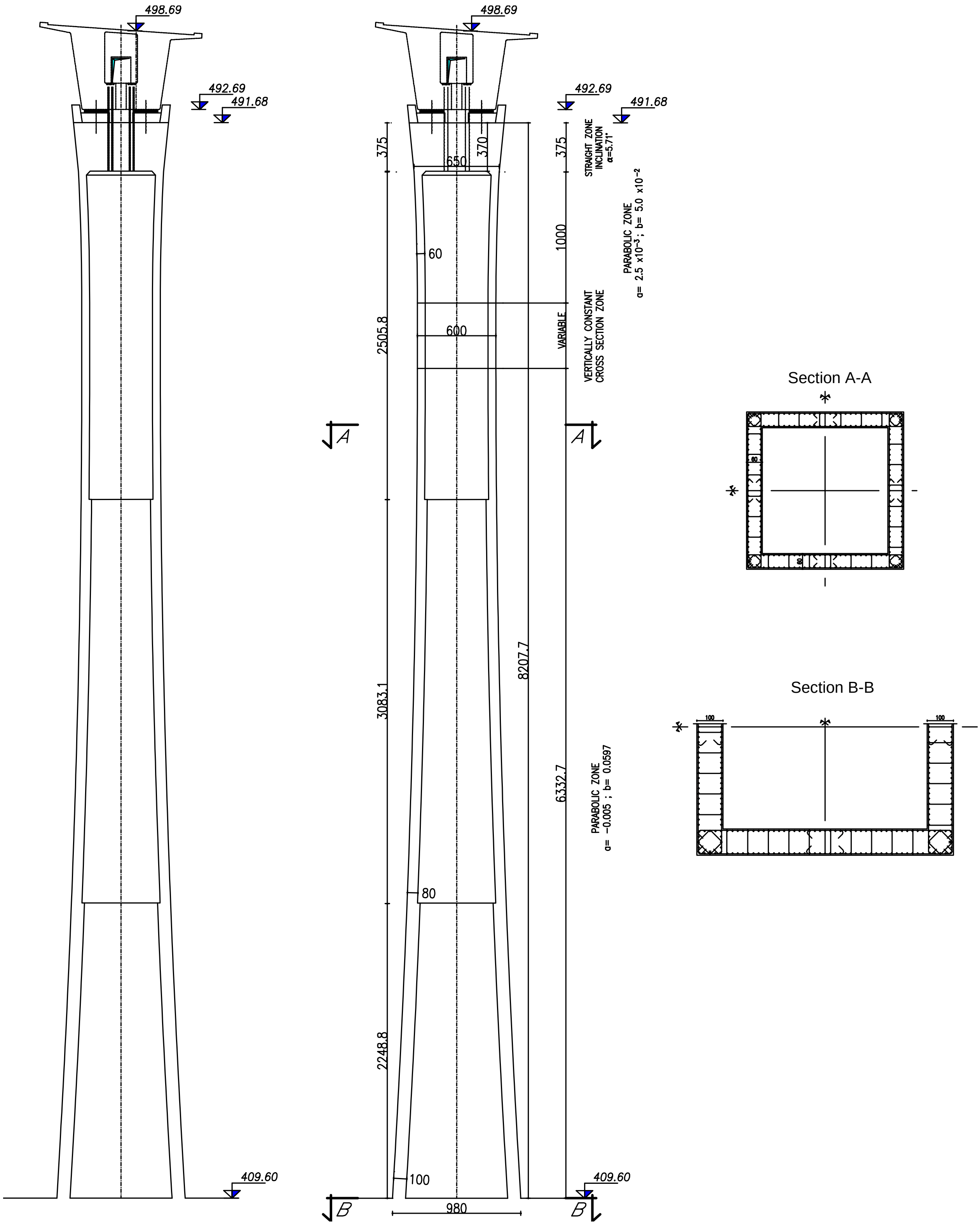

Figure 12 – Pier geometry and bar arrangement

In each pier, the critical section is located at a distance of 53.30 m from the base, where both the thickness and reinforcement undergo a change.

Applying the safety format to that section in the four analysis, we can obtain the results summarized in Tables 6 and 7, where $\gamma_{Gu} = \gamma_{Qu}$ represents the multiplier of the serviceability loads to be carried at failure.

Table. 6 – Safety format: γ_{Gl}

Pier depth [m]	$\frac{\gamma_{Gu}}{\gamma_{Gl}}$	Critical section		Top section		$\gamma_G(E(\gamma_{gl})/\gamma_{Rd})$	Gain Γ
		$N(\gamma_{Gl})^{*}$ [kN×10³]	$M(\gamma_{Gl})^{**}$ [kNm×10³]	*N Safety* [kNm×10³]	*M Safety* [kNm×10³]		
82	2.45	151	242	134	74.9	2.42	62%
87	2.15	137	243	119	66.5	2.15	43%
92	1.85	122	233	103	57.6	1.86	24%
97	1.58	103	218	86	48.1	1.56	4%

Where:

$$^{*}N(\gamma_{Gl}) = N\left(\frac{\gamma_{Gu}\cdot G + \gamma_{Qu}\cdot Q}{\gamma_{Gl}}\right) = N\left(\frac{\gamma_{Gu}\cdot (G+Q)}{\gamma_{Gl}}\right)$$

$$^{**}M(\gamma_{Gl}) = M\left(\frac{\gamma_{Gu}\cdot G + \gamma_{Qu}\cdot Q}{\gamma_{Gl}}\right) = M\left(\frac{\gamma_{Gu}\cdot (G+Q)}{\gamma_{Gl}}\right)$$

Table. 7 – Safety format: γ_{gl}

Pier depth [m]	$\frac{\gamma_{Gu}}{\gamma_{gl}}$	Critical section					
		$N(\gamma_{gl})^{*}$ [kN×10³]	$M(\gamma_{gl})^{**}$ [kNm×10³]	$N(\gamma_{gl})/\gamma_{Rd}$ [kNm×10³]	$M(\gamma_{gl})/\gamma_{Rd}$ [kNm×10³]	*N Safety* [kNm×10³]	*M Safety* [kNm×10³]
82	2.59	159	260	150	245	152	243
87	2.28	146	262	138	247	139	246
92	1.96	129	251	122	237	123	236
97	1.67	110	234	104	221	104	220

Where:

$$^{*}N(\gamma_{gl}) = N\left(\frac{\gamma_{Gu}\cdot G + \gamma_{Qu}\cdot Q}{\gamma_{gl}}\right) = N\left(\frac{\gamma_{Gu}\cdot (G+Q)}{\gamma_{gl}}\right)$$

$$^{**}M(\gamma_{gl}) = M\left(\frac{\gamma_{Gu}\cdot G + \gamma_{Qu}\cdot Q}{\gamma_{gl}}\right) = M\left(\frac{\gamma_{Gu}\cdot (G+Q)}{\gamma_{gl}}\right)$$

Pier depth [m]	Top section		$\gamma_G(E(\gamma_{gl})/\gamma_{Rd})$	Gain Γ
	N Safety [kNm*10³]	*M Safety* [kNm*10³]		
82	135	75.4	2.44	63%
87	121	67.6	2.19	46%
92	104	58.1	1.88	25%
97	87	48.6	1.57	5%

The gain is, as before, the incremental percentage of γ between the nonlinear and the linear solutions :

$$\Gamma = \frac{\gamma_{Gu} - 1.5}{1.5} = \frac{\gamma_{Qu} - 1.5}{1.5}$$

In tables 6,7,8 N Safety and M Safety represent the top-pier actions after the application of the safety format on the critical section.

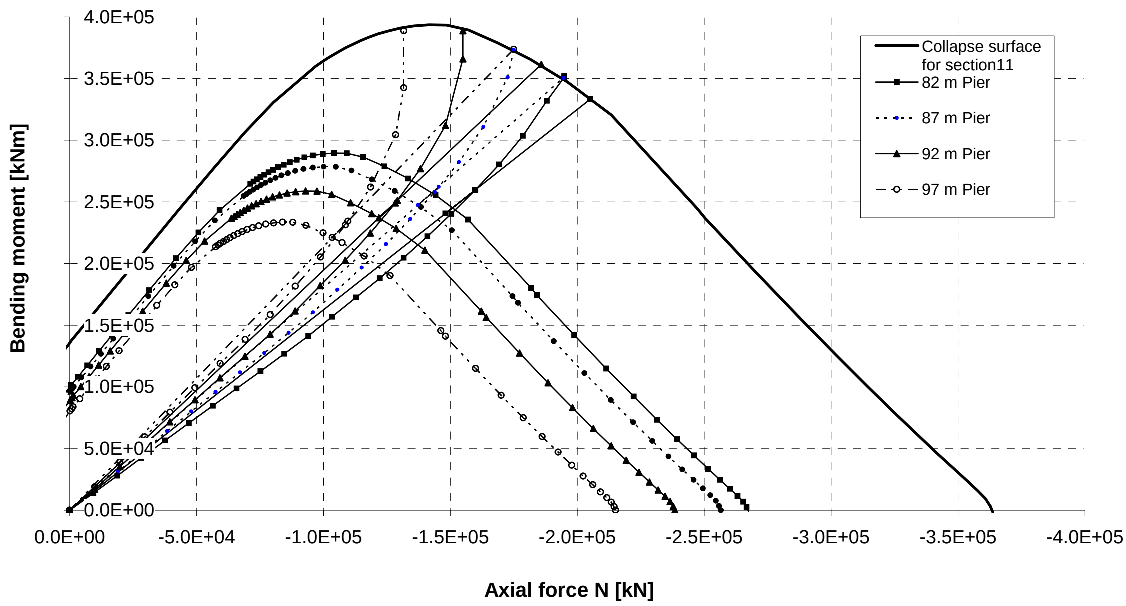

Figure 13 – Safety format: γ_{gl}

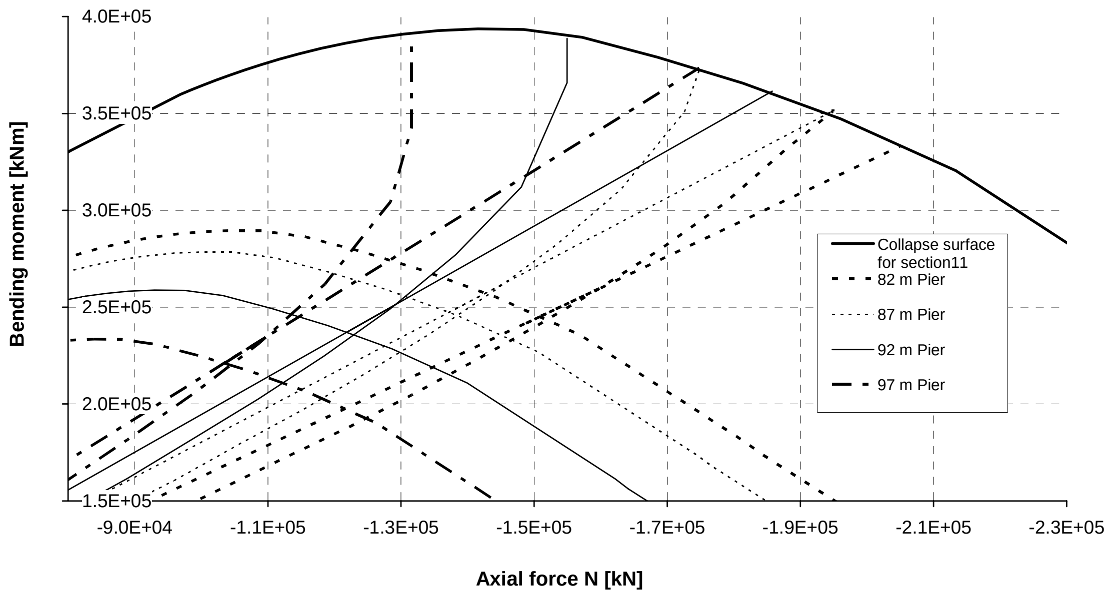

Figure 14 – Safety format: γ_{gl} – enlargement of the most interesting region of the homothetic curves

It is clear that the complete procedure including model uncertainties gives only a modest increment in the gain, because the stress path does not trigger a large nonlinear behaviour.

As a conclusion, the presented approach has been compared to the method suggested in EN 1992-1-1, 5.8.6 General method. The suggested material values for nonlinear analysis are: $f_{cd} = f_{ck} /1.5$, $E_{cd} = E_{cm}/1.2$; $f_{yd} = f_{yk} / 1.15$, $\varepsilon_{ud} = 75/1000$, $k \leq (f_t/f_y)_k = 1.15$.

Table. 8 – Comparison between EN 1992-1-1 and current proposal (γ_{gl})

	Top section EC2		Top section		Gain	
Pier depth [m]	*N* [kN×10³]	*M* [kNm×10³]	*N Safety* [kNm×10³]	*M Safety* [kNm×10³]	*% on N*	*% on M*
82	136	77.0	135	75.4	-1%	-2%
87	123	68.7	121	67.6	-2%	-2%
92	108	60.4	104	58.1	-4%	-4%
97	93.3	52.1	87	48.6	-7%	-7%

It is clear from the above comparison that the two methods give similar results and then the 5.8.6 General method of EN 1992-1-1 can be maintained as an alternative.

8 NONLINEAR ANALYSIS OF A CONTINUOUS DEEP BEAM

In this last case, a symmetric, 2-spans, well documented ([8], beam 3/1.5T1) continuous deep beam has been considered.

The nonlinear analysis was carried out by using ADINA non-linear code, for this purpose modified with an implemented definition of concrete strength [9, 10]; Fig. 15 shows the mesh of a half beam and the critical elements that govern the structural behaviour. In the same figure, the load-displacement curve for the mid span node A is given, and the fitting of the test results is quite good.

During the incremental analysis initially crushed the first critical element, but the structure was able to carry further load increments up to the crushing of the second one, in which case the model was unable to reach the equilibrium for further load increments. This last step has been considered as the final point of the internal actions path.

Fig. 16 shows the resisting interaction envelope σ_x, σ_y, τ_{xy} for the critical-second element, drawn in agreement with [10], and the internal action path in the same element, up to the intersection with the resisting envelope.

Figure 17 illustrates the procedure for the application of the safety format in a vectorial combination of internal action, by means of definition of a safety interaction surface, derived by the limit one by a linear transformation referred to the origin of the axes.

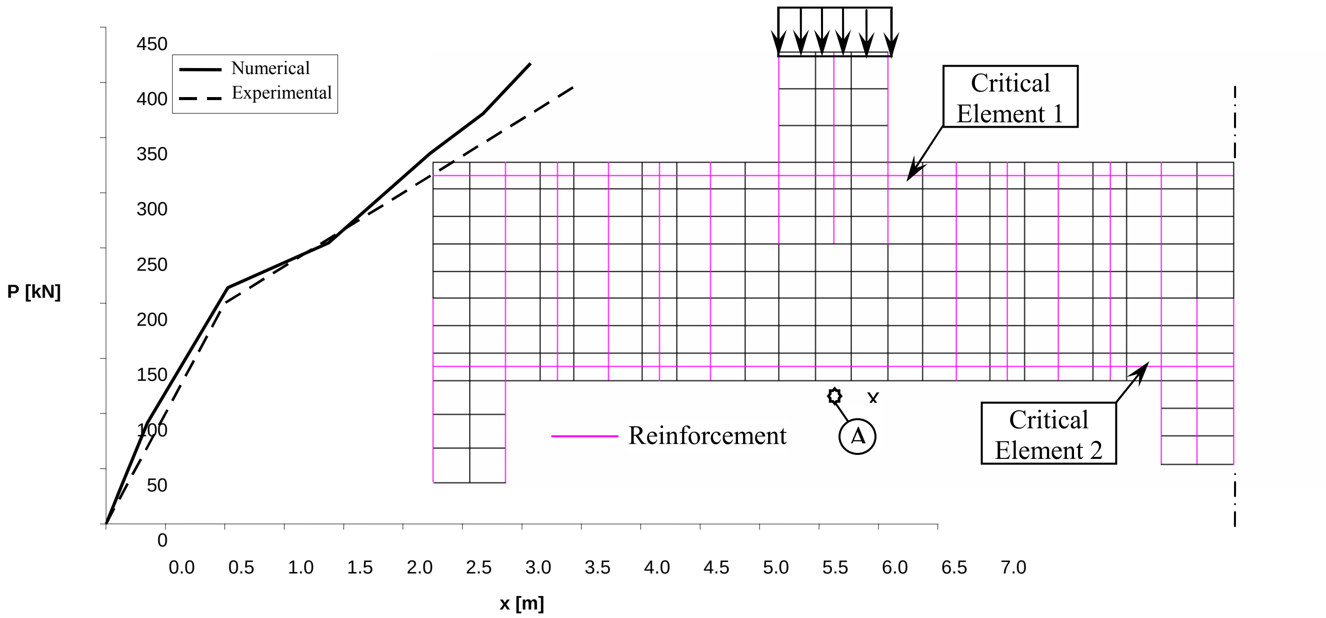

Figure 15 – R/C deep beam: FE half mesh (right) load-displacement curve of point A (left)

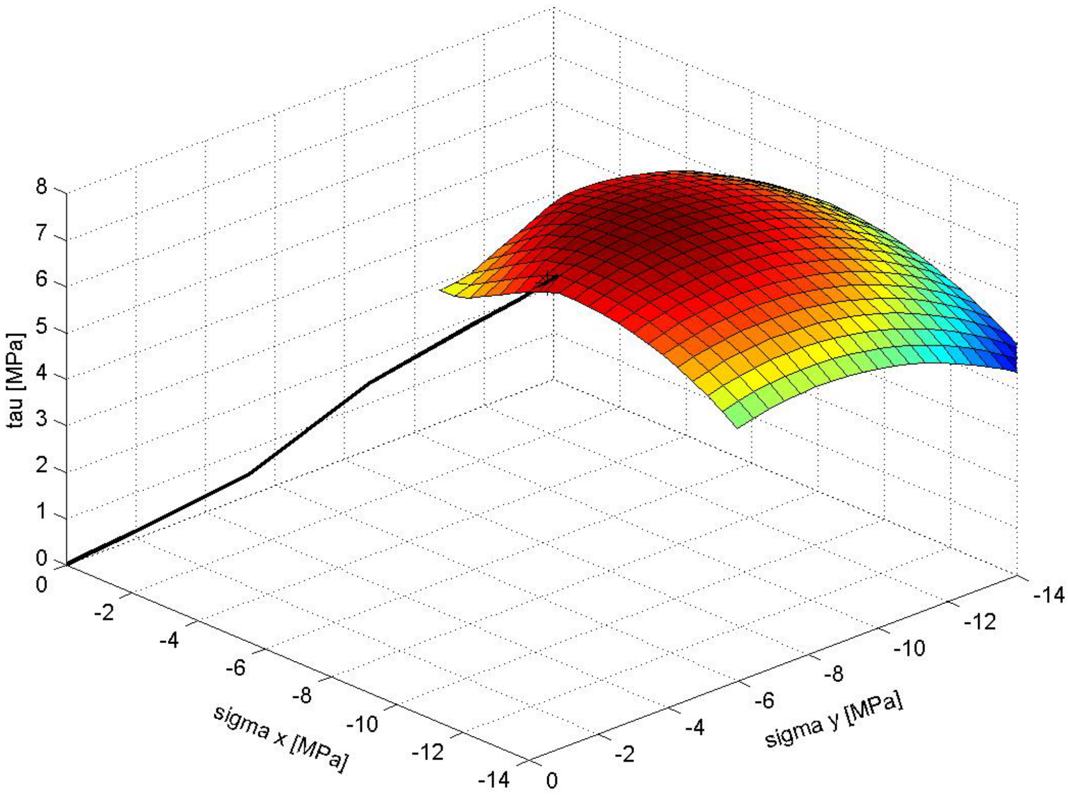

Figure 16 – Resisting interaction surface σ_x, σ_x, τ_{xy}

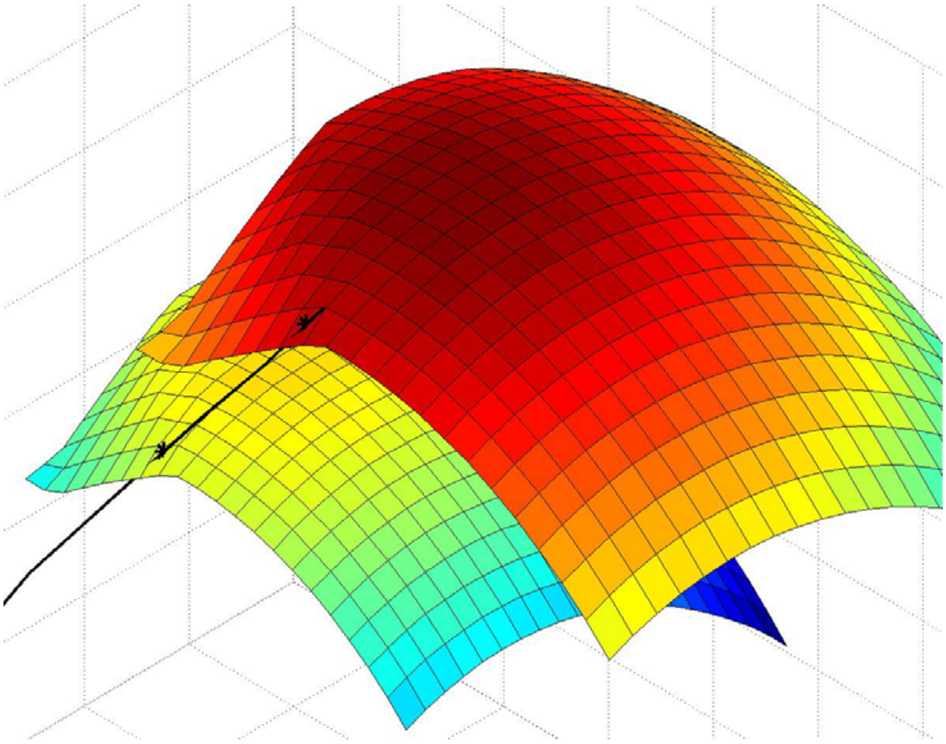

Figure 17 – Application of safety format in the vectorial space of internal actions

In practice, in agreement with the procedure outlined in [5], by applying equation (8), the following steps have been performed:

- individuation along the internal-action path of the internal-action set corresponding to q_{ud}/γ_{gl} ;
- linearization of this set with respect to the origin, by dividing it by γ_{Rd};
- definition by homotethy of the interaction envelope passing trough the new linearized set of internal actions;
- determination of the intersection of the internal-action path with the second interaction envelope and then of the limit set of internal action corresponding to the required safety level.

In the case considered here, the following sets of internal actions and corresponding values of the applied load (q_{ud} and q_{max}) were found:

- $q_{ud} = 404.0\,kN$, $\sigma_x = -8.47\,MPa$, $\sigma_y = -5.77\,MPa$, $\tau = 6.99\,MPa$
- $q_{max} = 319.5\,kN$, $\sigma_x = -6.82\,MPa$, $\sigma_y = -4.75\,MPa$, $\tau = 5.69\,MPa$

In this case, due to the limited nonlinearity in the internal action path of the critical second element, the ratio q_{ud}/q_{max} (1.26) practically coincides with the corresponding one with $\gamma_{Rd} = 1.0$ and $\gamma_{Gl} = 1.27$.

9 CONCLUSIONS

A safety format for a nonlinear analysis coherent with the semi probabilistic approach in R/C structures has been illustrated. The proposal is able to take into account also model uncertainties not explicitly considered when modelling the structure. The safety format is applied to linear and two dimensional elements, taking account of second order effects, where relevant.

Numerical examples demonstrate that model uncertainties need to be taken into account in linear elements, but their effects can be generally disregarded in 2D or 3D elements due to the high internal redundancy.

The definition of the level to which the linearization should be performed is suggested in the paper, essentially on the base of engineering judgement; but an improved definition of γ_{Rd} values may need further analyses based on sensitivity approach of the most influencing parameters.

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